ACADEMIC WORLD SCHOOL™

BEMETARA

SUMMER VACATION ASSIGNMENT

SESSION 2020-21

CLASS: XII

SUBJECT- MATHEMATICS

General Instructions

(i) *This assignment contains 40 questions*

(ii) *Q.1 to Q.10 is Short Answer Type.*

(iii) *Q.11 to Q.25 is Long Answer Type I.*

(iv) *Q.26 to Q.40 is Long Answer Type II.*

1. Find the value of x, y, z and w which satisfy the matrix equation: $\begin{bmatrix} x+2 & 2y+x \\ z-4 & 4w-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & w \end{bmatrix}$

2. Simplify: $cos\theta \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} + sin\theta \begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix}$.

3. If $X_{m\times 3} Y_{p\times 4} = Z_{2\times b}$, for three matrices X, Y and Z, find the values of m, p and b.

4. If A is a square matrix such that $A^2 = A$, then write the value of $(I+A)^3 - 3A$.

5. If matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is the square root of the 2×2 identity matrix, then find the relation between a, b and c.

6. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

7. Evaluate the determinant: $\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$

8. Without expanding, show that: $\begin{vmatrix} cosec^2\theta & cot^2\theta & 1 \\ cot^2\theta & cosec^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$

9. If the value of third order determinant is 12, then find the value of the determinant formed by its co-factors.

10. If $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then find the value of $|adjA|$.

11. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find the values of x and y such that $A^2 - xA + yI = O$, where I is a 2×2 unit matrix and O is a 2×2 zero matrix.

12. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I = O$.

13. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the values of a and b.

14. If $f(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $f(x)f(y) = f(x+y)$.

15. Express the following matrix as the sum of a symmetric and a skew-symmetric matrix and verify your result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

16. For the matrices A and B, verify that $(AB)' = B'A'$, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$.

17. If the matrix $\begin{bmatrix} -5 & x-y & 6 \\ 2 & 0 & 4 \\ x+y & 2 & 1 \end{bmatrix}$ is symmetric, find $3x + y - 5z$.

18. Find the integral values of x, if $\begin{bmatrix} x & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} x & 4 & -1 \end{bmatrix}' = O$.

19. If $A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$, verify that $AA' = I$.

20. If $\begin{bmatrix} cos\frac{2\pi}{5} & -sin\frac{2\pi}{5} \\ sin\frac{2\pi}{5} & cos\frac{2\pi}{5} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find the least positive integral value of k.

21. Three schools A, B and C organized a Mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs. 100 and Rs. 50 each. The numbers of articles sold are given:

School→ Article↓	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.

22. To promote the making of toilets for women, an organization tried to generate awareness through (i) house calls, (ii) letters, and (iii) announcements. The cost of each mode per attempt is given as: (i) Rs. 50 (ii) Rs. 20 (iii) Rs. 40

The number of attempts made in three villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately, using matrices.

23. A trust caring for handicapped children gets Rs. 30,000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust get from the bank to get a total of Rs. 1,800 every month?

24. Show that the points $(a+5, a-4), (a-2, a+3)$ and (a, a) do not lie on a straight line for any value of a.

25. Find the equation of a line joining the points $A(-3,2)$ and $B(4,0)$. Also find the value of α, if $ar(ABC) = 7\ sq.units$, where point C is $(0, \alpha)$.

Using the properties of determinants, prove that:

26. $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

27. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

28. If a, b and c are real numbers and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0.$

Show that either $a+b+c=0$ or $a=b=c$.

29. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

30. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x)-f(x)$.

31. If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then prove that $\Delta + \Delta_1 = 0$.

32. If $a+b+c=2s$, prove that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$.

33. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$.

34. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the following system of equations:

$$2x - 3y + 5z = 16;\ 3x + 2y - 4z = -4; x + y - 2z = -3$$

35. Solve the system of the following equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4;\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1;\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

36. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Verify that $BA = 6I$, use the result to solve the system $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$.

37. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

38. A school wants to award its students for the values of honesty, Regularity and Hard work with a total cash award of Rs. 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs. 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

39. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in N$.

40. The management committee of a residential colony decided to award some of its members (say x) for Honesty, some (say y) for Helping others and some others (say z) for Supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for Cooperation and Supervision added to two times the number of awardees for Honesty is 33. If the sum of the number of awardees for Honesty and Supervision is twice the number of awardees for Helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, Honesty, Cooperation and Supervision, suggest one more value which the management of the colony must include for awards.

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