ACADEMIC WORLD SCHOOL™
BEMETARA

Class- VI
Subject- Mathematics

Chapter- 1 Knowing our Numbers

Number

A number is a mathematical object used to count, measure, and also label.

Comparing Numbers

1. Compare 4978 and 5643…….

5643 is greater as the digit at the thousands place in 5643 is greater than that in 4978.

2. Compare 9364,8695,8402 and 7924

9364 is the greatest as it has the greatest digit at the thousands place in all the numbers.

Whereas 7924 is the smallest as it has the smallest digit at the thousands place in all the numbers.

3. Special case

Compare 56321 and 56843

Here, we will start by checking the thousands place. As the digit 5 at ten thousand place is same so we will move forward and see the thousands place. The digit 6 is also same so we will still move on further to check the hundreds place.

The digit at the hundreds place in 56843 is greater than that in 56321

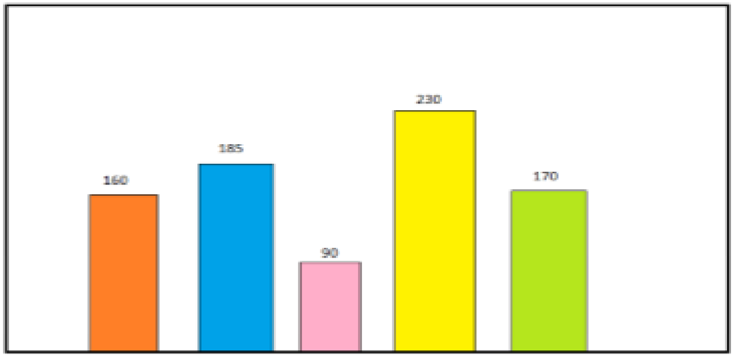
Thus 56843 is greater than 56321

Proper Order

- If we arrange the numbers from the smallest to the greatest then it is said to be an **Ascending order**.
- If we arrange the numbers from the greatest to the smallest then it is said to be **Descending order**.

Example

Arrange the following heights in ascending and descending order.



Ascending order – 90 < 160 < 170 < 185 < 230

Descending order – 230 >185 >170 > 160 > 90

Number Formations

Form the largest and the smallest possible numbers using 3,8,1,5 without repetition

Largest number will be formed by arranging the given numbers in descending order – 8531

The smallest number will be formed by arranging the given numbers in ascending order – 1358

Introducing 10,000

99 is the greatest 2-digit number.

999 is the greatest 3-digit number

9999 is the greatest 4-digit number

Observation

- If we add 1 to the greatest single digit number then we get the smallest 2-digit number

  (9 + 1 = 10)

- If we add 1 to the greatest 2- digit number then we get the smallest 3-digit number

  (99 + 1 = 100)

- If we add 1 to the greatest 3- digit number then we get the smallest 4-digit number

  (999 + 1 = 1000)

Moving forward, all the above situations are same as adding 1 to the greatest 4-digit number is the same as the smallest 5-digit number. (9999 + 1 = 10,000), and it is known as **ten thousand**.

Place Value

It refers to the positional notation which defines a digits position.

Example

6931

Here, 1 is at one's place, 3 is at tens place, 9 is at hundreds place and 6 is at thousands place

Expanded form

It refers to expand the number to see the value of each digit.

Example

6821 = 6000 + 800 + 20 + 1= 6 × 1000 + 8 × 100 + 2 × 10 + 1×1

Introducing 1,00,000

As above pattern if we add 1 to the greatest 5-digit number then we will get the smallest 6-digit number

(99,999 + 1 = 1,00,000)

This number is called **one lakh**.

Larger Numbers

To get the larger numbers also, we will follow the same pattern.

We will get the smallest 7-digit number if we add one more to the greatest 6-digit number, which is called **Ten Lakh**.
Going forward if we add 1 to the greatest 7-digit number then we will get the smallest 8-digit number which is called **One Crore**.

Remark

1 hundred = 10 tens

1 thousand = 10 hundred = 100 tens

1 lakh = 100 thousands = 1000 hundreds

1 crore = 100 lakhs = 10,000 thousands

Pattern

9 + 1 = 10

99 + 1 = 100

999 + 1 = 1000

9,999 + 1 = 10,000

99,999 + 1 =1,00,000

9,99,999 + 1 = 10,00,000

99,99,999+1=1,00,00,000

Reading and Writing Large Numbers

Crores		Lakhs		Thousands		Ones		
Ten Crores (TC)	Crores (C)	Ten Lakhs (TL)	Lakhs (L)	Ten Thousands (TTh)	Thousands (Th)	Hendreds (H)	Tens (T)	Ones (O)
(10, 00, 00, 000)	(1,00,00,000)	(10, 00, 000)	(1,00,000)	(10,000)	(1000)	(100)	(10)	(1)

We can identify the digits in ones place, tens place and hundreds place in a number by writing them under the tables O, T and H.

AS:

Example

Represent the number 5, 21, 05, 747

Use of Commas

We use commas in large numbers to ease reading and writing. In our **Indian System of Numeration,** we use ones, tens, hundreds, thousands and then lakhs and crores.

We use the first comma after hundreds place which is three digits from the right. The second comma comes after two digits i.e. five digits from the right. The third comma comes after another two digits which is seven digits from the right.

Example

5,44,12,940

Remark: We do not use commas while writing number names

International System of Numeration

Millons			Thousands			Ones		
Hundred Million	Ten Million	Million	Hundred Thousands	Ten Thousands	Thousands	Hundred	Tens	Ones
100,000,000	10,000,000	1,000,000	100,000	10,000	1,000	100	10	1

Example

341,697,832

Expanded form: 3 x 100,000,000 + 4 x 10,000,000 + 1 x 1,000,000 + 6 x 100,000 + 9 x 10,000 + 7 x 1,000 + 8 x 100 + 3 x 10 + 2 x 1

Remark: If we have to express the numbers larger than a million then we use a billion in the International System of Numeration:

1 billion = 1000 million

EXERCISE 1.1

1. Fill in the blanks:
   (a) 1 lakh = _______ ten thousand.
   (b) 1 million = _______ hundred thousand.
   (c) 1 crore = _______ ten lakh.
   (d) 1 crore = _______ million.
   (e) 1 million = _______ lakh.
2. Place commas correctly and write the numerals:
   (a) Seventy three lakh seventy five thousand three hundred seven.
   (b) Nine crore five lakh forty one.
   (c) Seven crore fifty two lakh twenty one thousand three hundred two.
   (d) Fifty eight million four hundred twenty three thousand two hundred two.
   (e) Twenty three lakh thirty thousand ten.
3. Insert commas suitably and write the names according to Indian System of Numeration :
   (a) 87595762 (b) 8546283 (c) 99900046 (d) 98432701
4. Insert commas suitably and write the names according to International System of Numeration :
   (a) 78921092 (b) 7452283 (c) 99985102 (d) 48049831

Large Numbers in Practice

10 millimetres = 1 centimetre

1 meter = 100 centimetres = 1000 millimetres

1 kilometre = 1000 meters

1 kilogram = 1000 grams.

1 gram = 1000 milligrams

. 1 litre = 1000 millilitres

1 litre = 1000 millilitres

Let's Solve Some Problems

.

Example: 2

There are 8797 children in a town, 6989 go to school. How many children do not go to school?

Solution:

Total children = 8797

Children who go to school = 6989

Children who do not go to school = 8797 – 6989= 1808

Thus 1808 children of the town do not go to school.

Example: 3

There are 24 folders and each has 56 sheets of paper inside them. How many sheets of paper are there altogether?

Solution:

$$\begin{array}{r} 56 \\ \times 24 \\ \hline 224 \\ +1120 \\ \hline 1344 \\ \hline \end{array}$$

Thus there are 1344 sheets of paper altogether.

Example: 4

$5,876 is distributed equally among 26 men. How much money will each person get?

Solution:

Money received by 26 men = 5876

Money received by one man = 5876 ÷ 26

$$\begin{array}{r} 226 \\ 26\overline{)5876} \\ -52\downarrow\downarrow \\ \hline 67\downarrow \\ -52 \\ \hline 156 \\ -156 \\ \hline 0 \end{array}$$

Thus, each man got $226.

EXERCISE 1.2

1. A book exhibition was held for four days in a school. The number of tickets sold at the counter on the first, second, third and final day was respectively 1094, 1812, 2050 and 2751. Find the total number of tickets sold on all the four days.
2. Shekhar is a famous cricket player. He has so far scored 6980 runs in test matches. He wishes to complete 10,000 runs. How many more runs does he need?
3. In an election, the successful candidate registered 5,77,500 votes and his nearest rival secured 3,48,700 votes. By what margin did the successful candidate win the election?
4. Kirti bookstore sold books worth ₹ 2,85,891 in the first week of June and books worth ₹ 4,00,768 in the second week of the month. How much was the sale for the

two weeks together? In which week was the sale greater and by how much?

5. Find the difference between the greatest and the least 5-digit number that can be written using the digits 6, 2, 7, 4, 3 each only once.
6. A machine, on an average, manufactures 2,825 screws a day. How many screws did it produce in the month of January 2006?
7. A merchant had ₹ 78,592 with her. She placed an order for purchasing 40 radio sets at ₹ 1200 each. How much money will remain with her after the purchase?
8. A student multiplied 7236 by 65 instead of multiplying by 56. By how much was his answer greater than the correct answer? (**Hint:** Do you need to do both the multiplications?)
9. To stitch a shirt, 2 m 15 cm cloth is needed. Out of 40 m cloth, how many shirts can be stitched and how much cloth will remain?

   (Hint: convert data in cm.)
10. Medicine is packed in boxes, each weighing 4 kg 500g. How many such boxes can be loaded in a van which cannot carry beyond 800 kg?
11. The distance between the school and a student's house is 1 km 875 m. Everyday she walks both ways. Find the total distance covered by her in six days.
12. A vessel has 4 litres and 500 ml of curd. In how many glasses, each of 25 ml capacity, can it be filled?

Estimation

It is a rough calculation of value. We use estimations when we have to deal with large numbers and to do the quick calculations.

Estimating to the nearest tens by rounding off

7$\underline{3}$8 → 740

If the digit in the ones places is:

5 or higher, round tens place up

4 or lower, leave tens place as is

Firstly, to estimate we need to see where does the number lies.

Here 38 lies between 30 and 40

Secondly, we will see if it is 5 or higher.

Yes, it is higher than 5 i.e. 3**8**

Thus, the number 7**38** is rounded off to 7**40**.

Estimating to the nearest hundreds by rounding off

Round off the number **867** nearest to the hundreds.

It lies between 800 and 900

Now we have to check for tens place. If it is greater than 50 then we will round it off to the upper side and if it is less than 50 then we will round it off on the lower side.

It is 67, which is greater than 50 and is closer to 900.

Thus 867 is rounded off to 900

Estimating to the nearest thousands by rounding off

The Numbers from 1 to 499 are rounded off to 0 as they are nearer to 0, and the numbers from 501 to 999 are rounded off to 1000 as they are nearer to 1000

And 500 is always rounded off to 1000.

Example

Round off the number 7690 nearest to thousands.

It lies between 7000 and 8000

And is closer to 8000

Thus 7690 is rounded off to 8000.

To estimate sum or difference

Estimate: 3,210 + 12,884

Solution 3,210 will be rounded off to 3000.

12,884 will be rounded off to 13000.

3000+ 13000

Estimated solution = 16000

Actual solution = 3,210 + 12 884= 16,094

To estimate products

Estimate: 73 × 18

Solution 73 will be rounded off to 70

18 will be rounded off to 20

70 × 20

Estimated solution = 1400

Actual solution = 73 × 18= 1314

Using Brackets

We use brackets to indicate that the numbers inside should be treated as a different number thus the bracket should be solved first.

Example

8 + 2 = 8 + 10 = 18

Whereas if we use brackets

(8 + 2) × 5 = 10 = 50

Expanding brackets

Brackets help in the systematic calculation.

Example

$3 \times 109 = 3 \times (100 + 9)$

$= 3 \times 100 + 3 \times 9 = 300 + 27 = 327$

EXERCISE 1.3

1. Estimate each of the following using general rule:

   (a) 730 + 998 (b) 796 – 314 (c) 12,904 +2,888 (d) 28,292 – 21,496

   Make ten more such examples of addition, subtraction and estimation of their outcome.

2. Give a rough estimate (by rounding off to nearest hundreds) and also a closer estimate (by rounding off to nearest tens) :

   (a) 439 + 334 + 4,317 (b) 1,08,734 – 47,599 (c) 8325 – 491

   (d) 4,89,348 – 48,365

   Make four more such examples.

3. Estimate the following products using general rule:

   (a) 578 × 161 (b) 5281 × 3491 (c) 1291 × 592 (d) 9250 × 29

   Make four more such examples.

Roman Numerals

Numbers in this system are represented by combinations of letters from the Latin alphabets.

1 =	I	40 =	XL
2 =	II	50 =	L
3 =	III	60 =	LX
4 =	IV	70 =	LXX
5 =	V	80 =	LXXX
6 =	VI	90 =	XC
7 =	VII	100 =	C
8 =	VIII	101 =	CI
9 =	IX	150 =	CL
10 =	X	200 =	CC
20 =	XX	500 =	D
21 =	XXI	800 =	DCCC
30 =	XXX	1000 =	M

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Rules:

a. If we repeat a symbol, its value will be added as many times as it occurs:

Example

II is equal 2

XX is 20

XXX is 30.

b. We cannot repeat a symbol more than three times and some symbols like V, L and D can never be repeated.

c. If we write a symbol of lesser value to the right of a symbol of larger value then its value will be added to the value of the greater symbol.

VI = 5 + 1 = 6,   XII = 10 + 2 = 12 and LXV = 50 + 10 + 5 = 65.

d. If we write a symbol of lesser value to the left of a symbol of larger value then its value will be subtracted from the value of the greater symbol.

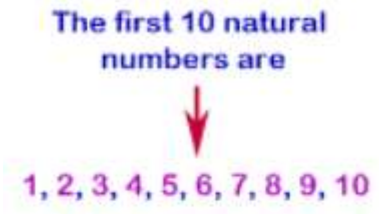
IV = 5 – 1 = 4, IX = 10 – 1 = 9 XL= 50 – 10 = 40, XC = 100 – 10 = 90.

e. The symbols V, L and D can never be subtracted so they are never written to the left of a symbol of greater value. We can subtract the symbol "I" from V and X only and the symbol X from L, M and C only.

CHAPTER -2 Whole Numbers

Natural Numbers

All the positive counting numbers starting from one are called **Natural Numbers**.



Predecessor and Successor

If we add 1 to any natural number, we get the next number, which is called the **Successor** of that number.

12 + 1 = 13

So 13 is the successor of 12.

If we subtract 1 from any natural number, we get the **predecessor** of that number.

12 – 1 = 11

So 11 is the predecessor of 12.

Remark: There is no predecessor of 1 in natural numbers.

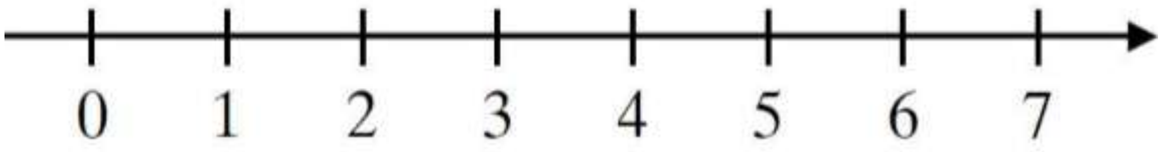
Whole Numbers

Whole numbers are the collection of natural numbers including zero. So, the zero is the predecessor of 1 in the whole numbers.

Number Line

To draw a number line, follow these steps-

- Draw a line and mark a point 0 on it.
- Now mark the second point to the right of zero and label it as 1.
- The distance between the 0 and 1 is called the unit distance.
- Now you can mark other points as 2, 3, 4 and so on with the unit distance.



This is the number line for the whole numbers.

1. The distance between two points

The distance between 3 and 5 is 2 units. Likewise, the distance between 1 and 6 is 5 units.

2. The greater number on the number line

The number on the right is always greater than the number on the left.

As number 5 is on the right of the number 2, Hence 5>2.

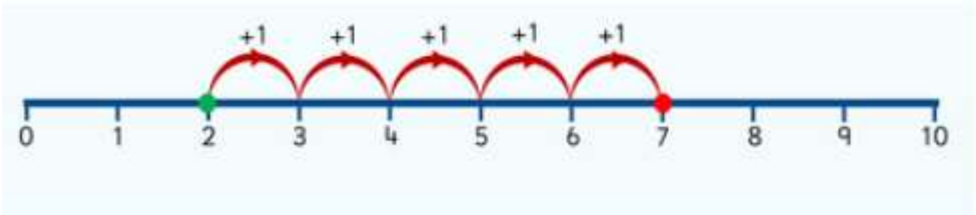
3. A smaller number on the number line

The number on the left of any number is always smaller than that number.

As number 3 is on the left of 7, so 3 < 7.

Addition on the Number Line

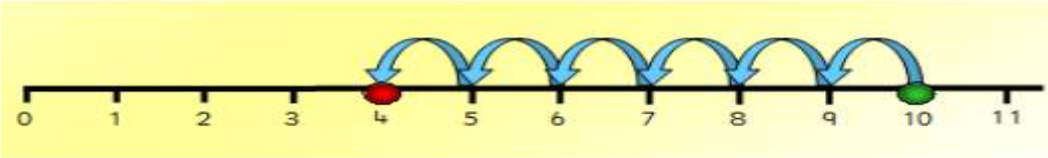
If we have to add 2 and 5, then start with 2 and make 5 jumps to the right. As our 5th jump is at 7 so the answer is 7.



The sum of 2 and 5 is 2 + 5 = 7

Subtraction on the Number Line

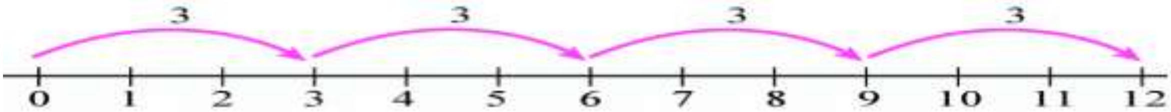
If we have to subtract 6 from 10, then we have to start from 10 and make 6 jumps to the left. As our 6th jump is at 4, so the answer is 4.



The subtraction of 6 from 10 is 10 – 6 = 4.

Multiplication on the Number Line

If we have to multiply 4 and 3, then Start from 0, make 4 jumps using 3 units at a time to the right, as you reach to 12. So, we say, 3 × 4 = 12.



using the
number line.

EXERCISE 2.1

1. Write the next three natural numbers after 10999.
2. Write the three whole numbers occurring just before 10001.
3. Which is the smallest whole number?
4. How many whole numbers are there between 32 and 53?
5. Write the successor of :

   (a) 2440701 (b) 100199 (c) 1099999 (d) 2345670
6. Write the predecessor of :

   (a) 94 (b) 10000 (c) 208090 (d) 7654321
7. In each of the following pairs of numbers, state which whole number is on the left of the other number on the number line. Also write them with the appropriate sign (>, <) between them.

   (a) 530, 503 (b) 370, 307 (c) 98765, 56789 (d) 9830415, 10023001
8. Which of the following statements are true (T) and which are false (F) ?

   (a) Zero is the smallest natural number. (b) 400 is the predecessor of 399.

   (c) Zero is the smallest whole number. (d) 600 is the successor of 599.

   (e) All natural numbers are whole numbers.

   (f) All whole numbers are natural numbers.

   (g) The predecessor of a two digit number is never a single digit number.

   (h) 1 is the smallest whole number.

   (i) The natural number 1 has no predecessor.

   (j) The whole number 1 has no predecessor.

   (k) The whole number 13 lies between 11 and 12.

   (l) The whole number 0 has no predecessor.

   (m) The successor of a two digit number is always a two digit number.

Properties of Whole Numbers

1. Closure Property

Two whole numbers are said to be closed if their operation is also the whole number.

Operation	Meaning	Example	Closed or not
Addition	Whole numbers are closed under addition as their sum is also a whole number.	2 + 5 = 7	Yes
Subtraction	Whole numbers are not closed under subtraction as their difference is not always a whole number.	9 – 2 = 7 2 – 9 = (-7) which is not a whole number.	No
Multiplication	Whole numbers are closed under multiplication as their product is also a whole number.	9 × 5 = 45	Yes
Division	Whole numbers are not closed under division as their result is not always a whole number.	5 ÷ 1 = 5 5 ÷ 2 =, not a whole number.	No

2. Commutative Property

Two whole numbers are said to be commutative if their result remains the same even if we swap the positions of the numbers.

Operation	Meaning	Example	Commutative or not
Addition	The addition is commutative for whole numbers as their sum remains the same even if we interchange the position of the numbers.	$2 + 5 = 7$ $5 + 2 = 7$	Yes
Subtraction	Subtraction is not commutative for whole numbers as their difference may be different if we interchange the position of the numbers.	$9 – 2 = 7$ $2 – 9 = (-7)$ which is not a whole number.	No
Multiplication	Multiplication is commutative for whole numbers as their product remains the same even if we interchange the position of the numbers.	$9 \times 5 = 45$ $5 \times 9 = 45$	Yes
Division	The division is not commutative for whole numbers as their result may be different if we interchange the position of the numbers.	$5 \div 1 = 5$ $1 \div 5 =$, not a whole number.	No

3. Associative Property

The two whole numbers are said to be associative if the result remains the same even if we change the grouping of the numbers.

Operation	Meaning	Example	Associative or not
Addition	The addition is associative for whole numbers as their sum remains the same even if we change the grouping of the numbers.	$3 + (2 + 5) = (3 + 2) + 5$ $3 + 7 = 5 + 5$ $10 = 10$	Yes
subtraction	Subtraction is not associative for whole numbers as their difference may change if we change the grouping of the numbers.	$8 – (10 – 2) \neq (8 – 10) – 2$ $8 - (8) \neq (-2) – 2$ $0 \neq (-4)$	No
Multiplication	Multiplication is associative for whole numbers as their product remains the same even if we change the grouping of the numbers.	$3 \times (5 \times 2) = (3 \times 5) \times 2$ $3 \times (10) = (15) \times 2$ $30 = 30$	Yes
Division	The division is not associative for whole numbers as their result may change if we change the grouping of the numbers.	$24 \div 3 \neq 4 \div 2$ $8 \neq 2$	No

4. Distributivity of Multiplication over Addition

This property says that if we have three whole numbers x, y and z, then

$x(y + z) = xy + xz$

Example

Evaluate 15×45

Solution

$15 \times 45 = 15 \times (40 + 5)$

$= 15 \times 40 + 15 \times 5$

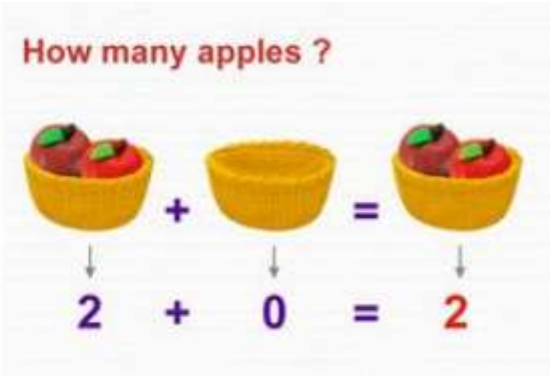
$= 600 + 75$

$= 675$

5. Identity for Addition

If we add zero to any whole number the result will the same number only. So zero is the additive identity of whole numbers.

$a + 0 = 0 + a = a$



This clearly shows that if we add zero apples to 2 apples we get the two apples only.

6. Identity for Multiplication

If we multiply one to any whole number the result will be the same whole number. So one is the multiplicative identity of whole numbers.

$$a \times (1) = a$$
$$1 \times (a) = a$$

EXERCISE 2.2

1. Find the sum by suitable rearrangement:

   (a) $837 + 208 + 363$ (b) $1962 + 453 + 1538 + 647$

2. Find the product by suitable rearrangement:

   (a) $2 \times 1768 \times 50$ (b) $4 \times 166 \times 25$ (c) $8 \times 291 \times 125$

   (d) $625 \times 279 \times 16$ (e) $285 \times 5 \times 60$ (f) $125 \times 40 \times 8 \times 25$

3. Find the value of the following:

   (a) $297 \times 17 + 297 \times 3$ (b) $54279 \times 92 + 8 \times 54279$

   (c) $81265 \times 169 - 81265 \times 69$ (d) $3845 \times 5 \times 782 + 769 \times 25 \times 218$

4. Find the product using suitable properties.

   (a) 738×103 (b) 854×102 (c) 258×1008 (d) 1005×168

5. A taxidriver filled his car petrol tank with 40 litres of petrol on Monday. The next day, he filled the tank with 50 litres of petrol. If the petrol costs ₹ 44 per litre, how much did he spend in all on petrol?

6. A vendor supplies 32 litres of milk to a hotel in the morning and 68 litres of milk in the evening. If the milk costs ₹ 45 per litre, how much money is due to the vendor per day?

7. Match the following:

   (i) $425 \times 136 = 425 \times (6 + 30 + 100)$ (a) Commutativity under multiplication.

   (ii) $2 \times 49 \times 50 = 2 \times 50 \times 49$ (b) Commutativity under addition.

   (iii) $80 + 2005 + 20 = 80 + 20 + 2005$ (c) Distributivity of multiplication over addition.

Patterns

Patterns are used for easy verbal calculations and to understand the numbers better.

We can arrange the numbers using dots in elementary shapes like triangle, square, rectangle and line.

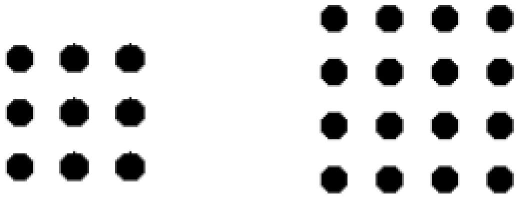
1. We can arrange every number using dots in a line

1 ●	2 ●●	3 ●●●	4 ●●●●	5 ●●●●●

2. We can arrange some numbers using a rectangle.

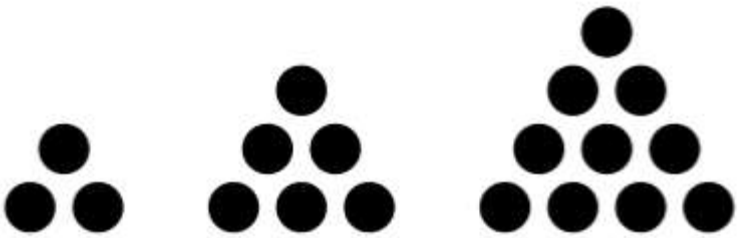
6 12

3. We can arrange some numbers using a square.



9 16

4. We can arrange some numbers using a triangle.



3 6 10

Use of Patterns

Patterns can be used to simplify the process.

1. 123 + 9 = 123 + 10 - 1 = 133 -1 = 132

123 + 99 = 123 + 100 – 1 = 223 – 1 = 222

2. 83 × 9 = 83 × (10-1) = 830 – 83 = 747

83 × 99 = 83 × (100-1) = 8300 – 83 = 8217

EXERCISE 2.3

1. Which of the following will not represent zero:

(a) $1 + 0$ (b) 0×0 (c) $\frac{0}{2}$ (d) $\frac{10-10}{2}$

2. If the product of two whole numbers is zero, can we say that one or both of them will be zero? Justify through examples.

3. If the product of two whole numbers is 1, can we say that one or both of them will be 1? Justify through examples.

4. Find using distributive property :

(a) 728×101 (b) 5437×1001 (c) 824×25 (d) 4275×125 (e) 504×35

5. Study the pattern :

$1 \times 8 + 1 = 9$ $\qquad$ $1234 \times 8 + 4 = 9876$

$12 \times 8 + 2 = 98$ $\qquad$ $12345 \times 8 + 5 = 98765$

$123 \times 8 + 3 = 987$

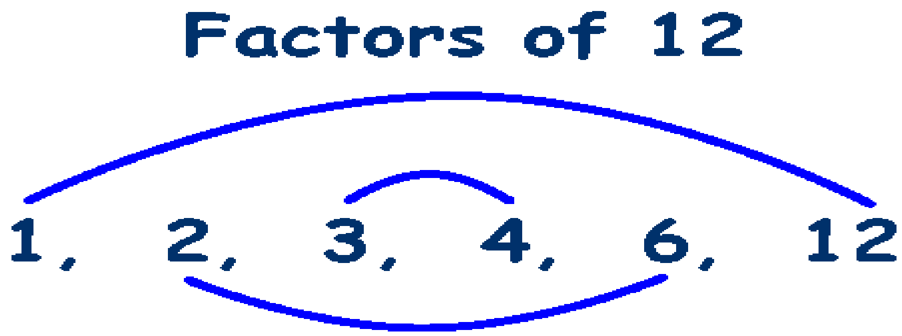
Write the next two steps. Can you say how the pattern works?

(Hint: $12345 = 11111 + 1111 + 111 + 11 + 1$).

Chapter-3 Playing with Numbers

Factors

The numbers which exactly divides the given number are called the **Factors** of that number.



As we can see that we get the number 12 by

1 × 12, 2 × 6, 3 × 4, 4 × 3, 6 × 2 and 12 ×1

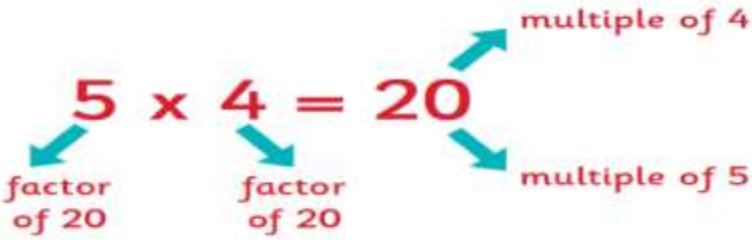
Hence,

1, 2, 3, 4, 6 and 12 are the factors of 12.

The factors are always less than or equal to the given number.

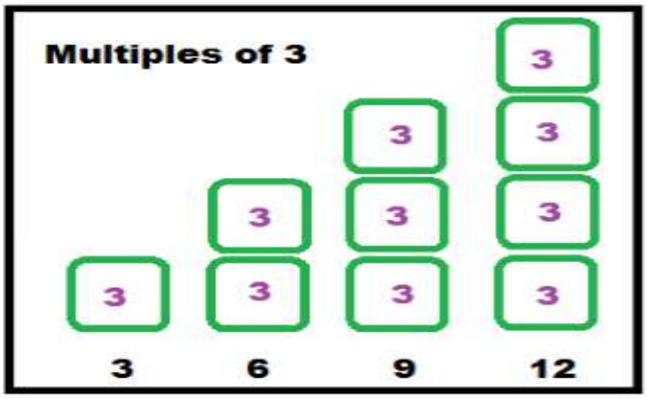
Multiples

If we say that 4 and 5 are the factors of 20 then 20 is the multiple of 4 and 5 both.



List the multiples of 3

Multiples are always more than or equal to the given number.



Some facts about Factors and Multiples

- 1 is the only number which is the factor of every number.
- Every number is the factor of itself.
- All the factors of any number are the exact divisor of that number.
- All the factors are less than or equal to the given number.
- There are limited numbers of factors of any given number.
- All the multiples of any number are greater than or equal to the given number.
- There are unlimited multiples of any given numbers.
- Every number is a multiple of itself.

EXERCISE 3.1

1. Write all the factors of the following numbers :

(a) 24	(b) 15	(c) 21
(d) 27	(e) 12	(f) 20
(g) 18	(h) 23	(i) 36

2. Write first five multiples of :

(a) 5 (b) 8 (c) 9

3. Match the items in column 1 with the items in column 2.

Column 1	Column 2
(i) 35	(a) Multiple of 8
(ii) 15	(b) Multiple of 7
(iii) 16	(c) Multiple of 70
(iv) 20	(d) Factor of 30

Perfect Number

If the sum of all the factors of any number is equal to the double of that number then that number is called a **Perfect Number**.

Perfect Number	Factors	Sum of all the factors
6	1, 2, 3, 6	12
28	1, 2, 4, 7, 14, 28	56
496	1, 2, 4, 8, 16, 31, 62, 124, 248, 496	992

Prime Numbers

The numbers whose only factors are 1 and the number itself are called the **Prime Numbers**.

Like 2, 3, 5, 7, 11 etc.

Composite Numbers

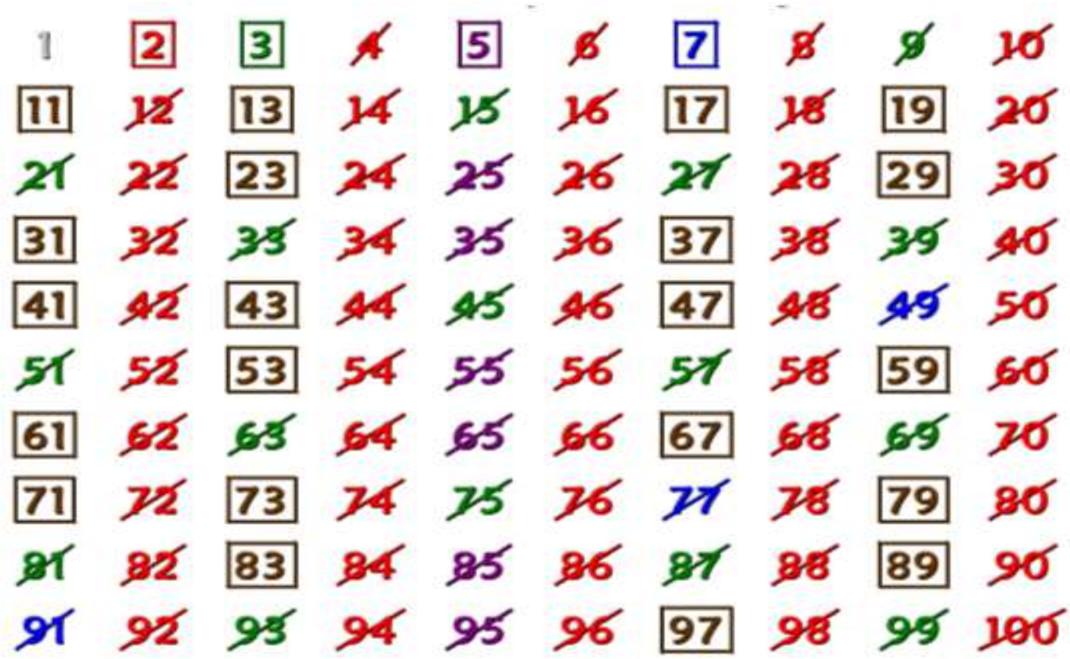
All the numbers with more than 2 factors are called composite numbers or you can say that the numbers which are not prime numbers are called **Composite Numbers**.

Like 4, 6, 8, 10, 12 etc.

Remark: 1 is neither a prime nor a composite number.

Sieve of Eratosthenes Method

This is the method to find all the prime numbers from 1 to 100.



Step 1: First of all cross 1, as it is neither prime nor composite.
Step 2: Now mark 2 and cross all the multiples of 2 except 2.
Step 3: Mark 3 and cross all the multiples of 3 except 3.
Step 4: 4 is already crossed so mark 5 and cross all the multiples of 5 except 5.
Step 5: Continue this process until all the numbers are marked square or crossed.

This shows that all the covered numbers are prime numbers and all the crossed numbers are composite numbers except 1.

Even and Odd Numbers

All the multiples of 2 are **even numbers**. To check whether the number is even or not, we can check the number at one's place. If the number at ones place is 0,2,4,6 and 8 then the number is even number.

The numbers which are not even are called **Odd Numbers**.

Remark: 2 is the smallest even prime number. All the prime numbers except 2 are odd numbers.

EXERCISE 3.2

1. What is the sum of any two (a) Odd numbers? (b) Even numbers?
2. State whether the following statements are True or False:
   (a) The sum of three odd numbers is even.
   (b) The sum of two odd numbers and one even number is even.
   (c) The product of three odd numbers is odd.
   (d) If an even number is divided by 2, the quotient is always odd.
   (e) All prime numbers are odd.
   (f) Prime numbers do not have any factors.
   (g) Sum of two prime numbers is always even.
   (h) 2 is the only even prime number.
   (i) All even numbers are composite numbers.
   (j) The product of two even numbers is always even.
3. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers upto 100.
4. Write down separately the prime and composite numbers less than 20.
5. What is the greatest prime number between 1 and 10?
6. Express the following as the sum of two odd primes.
   (a) 44 (b) 36 (c) 24 (d) 18
7. Give three pairs of prime numbers whose difference is 2.
   [Remark : Two prime numbers whose difference is 2 are called twin primes].
8. Which of the following numbers are prime?
   (a) 23 (b) 51 (c) 37 (d) 26
9. Write seven consecutive composite numbers less than 100 so that there is no prime number between them.
10. Express each of the following numbers as the sum of three odd primes:
    (a) 21 (b) 31 (c) 53 (d) 61
11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5
    (**Hint :** 3+7 = 10)
12. Fill in the blanks :
    (a) A number which has only two factors is called a _______.
    (b) A number which has more than two factors is called a _______.
    (c) 1 is neither _______ nor _______.
    (d) The smallest prime number is _______.
    (e) The smallest composite number is ______.
    (f) The smallest even number is _______.

Tests for Divisibility of Numbers

1. Divisibility by 2:

If there are any of the even numbers i.e. 0, 2, 4, 6 and 8 at the end of the digit then it is divisible by 2.

Example

Check whether the numbers 63 and 240 are divisible by 2 or not.

Solution:

1. The last digit of 63 is 3 i.e. odd number so 63 is not divisible by 2.

2. The last digit of 240 is 0 i.e. even number so 240 is divisible by 2.

2. Divisibility by 3:

A given number will only be divisible by 3 if the total of all the digits of that number is multiple of 3.

Example

Check whether the numbers 623 and 2400 are divisible by 3 or not.

Solution:

1. The sum of the digits of 623 i.e. 6 + 2 + 3 = 11, which is not the multiple of 3 so 623 is not divisible by 3.

2. The sum of the digits of 2400 i.e. 2 + 4 + 0 + 0 = 6, which is the multiple of 3 so 2400 is divisible by 3.

3. Divisibility by 4:

We have to check whether the last two digits of the given number are divisible by 4 or not. If it is divisible by 4 then the whole number will be divisible by 4.

Example

Check the number 23436 and 2582 are divisible by 4 or not.

Solution:

1. The last two digits of 23436 are 36 which are divisible by 4, so 23436 are divisible by 4.

2. The last two digits of 2582 are 82 which are not divisible by 4 so 2582 is not divisible by 4.

4. Divisibility by 5:

Any given number will be divisible by 5 if the last digit of that number is ‘0' or ‘5'.

Example

Check whether the numbers 2348 and 6300 are divisible by 5 or not.

Solution:

1. The last digit of 2348 is 8 so it is not divisible by 5.

2. The last digit of 6300 is 0 so it is divisible by 5.

5. Divisibility by 6:

Any given number will be divisible by 6 if it is divisible by 2 and 3 both. So we should do the divisibility test of 2 and 3 with the number and if it is divisible by both then it is divisible by 6 also.

Example

Check the number 342341 and 63000 are divisible by 6 or not.

Solution: 1. 342341 is not divisible by 2 as the digit at ones place is odd and is also not divisible by 3 as the sum of its digits i.e. 3 + 4 + 2 + 3 + 4 + 1 = 17 is also not divisible by 3.Hence 342341 is not divisible by 6.

2. 63000 is divisible by 2 as the digit at ones place is even and is also divisible by 3 as the sum of its digits i.e. 6 + 3 + 0 + 0 + 0 = 9 is divisible by 3.Hence 63000 is divisible by 6.

6. Divisibility by 7:

Any given number will be divisible by 7 if we double the last digit of the number and then subtract the result from the rest of the digits and check whether the remainder is divisible by 7 or not. If there is a large number of digits then we have to repeat the process until we get the number which could be checked for the divisibility of 7.

Example

Check the number 2030 is divisible by 7 or not.

Solution:

Given number is 2030

1. Double the last digit, 0 × 2 = 0

2. Subtract 0 from the remaining number 203 i.e. 203 – 0 = 203

3. Double the last digit, 3 × 2 = 6

4. Subtract 6 from the remaining number 20 i.e. 20 - 6 = 14

5. The remainder 14 is divisible by 7 hence the number 203 is divisible by 7.

7. Divisibility by 8:

We have to check whether the last three digits of the given number are divisible by 8 or not. If it is divisible by 8 then the whole number will be divisible by 8.

Example

Check whether the number 74640 is divisible by 8 or not.

Solution:

The last three digit of the number 74640 is 640.

As the number 640 is divisible by 8 hence the number 74640 is also divisible by 8.

8. Divisibility by 9:

Any given number will be divisible by 9 if the total of all the digits of that number is divisible by 9.

Example

Check whether the number 2320 and 6390 are divisible by 9 or not.

Solution:

1. The sum of the digits of 2320 is 2 + 3 + 2 + 0 = 7 which is not divisible by 9 so 2320 is not divisible by 9.

2. The sum of the digits of 6390 is 6 + 3 + 9 + 0 = 18 which is divisible by 9 so 6390 is divisible by 9.

9. Divisibility by 10:

Any given number will be divisible by 10 if the last digit of that number is zero.

Example

Check the number 123 and 2630 are divisible by 10 or not.

Solution:

1. The ones place digit is 3 in 123 so it is not divisible by 10.

2. The ones place digit is 0 in 2630 so it is divisible by 10.

EXERCISE 3.3

1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4; by 5; by 6; by 8; by 9; by 10 ; by 11 (say, yes or no):

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990	.....	.....	.....	.....	.....	.....	.....	.....	.....
1586	.....	.....	.....	.....	.....	.....	.....	.....	.....
275	.....	.....	.....	.....	.....	.....	.....	.....	.....
6686	.....	.....	.....	.....	.....	.....	.....	.....	.....
639210	.....	.....	.....	.....	.....	.....	.....	.....	.....
429714	.....	.....	.....	.....	.....	.....	.....	.....	.....
2856	.....	.....	.....	.....	.....	.....	.....	.....	.....
3060	.....	.....	.....	.....	.....	.....	.....	.....	.....
406839	.....	.....	.....	.....	.....	.....	.....	.....	.....

2. Using divisibility tests, determine which of the following numbers are divisible by 4; by 8:
(a) 572 (b) 726352 (c) 5500 (d) 6000 (e) 12159
(f) 14560 (g) 21084 (h) 31795072 (i) 1700 (j) 2150
3. Using divisibility tests, determine which of following numbers are divisible by 6:
(a) 297144 (b) 1258 (c) 4335 (d) 61233 (e) 901352
(f) 438750 (g) 1790184 (h) 12583 (i) 639210 (j) 17852
4. Using divisibility tests, determine which of the following numbers are divisible by 11:
(a) 5445 (b) 10824 (c) 7138965 (d) 70169308 (e) 10000001
(f) 901153
5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3 :
(a) ___ 6724 (b) 4765 ___ 2

Common Factors and Common Multiples

Example: 1

What are the common factors of 25 and 55?

Solution:

Factors of 25 are 1, 5.

Factors of 55 are 1, 5, 11.

Common factors of 25 and 55 are 1 and 5.

Example: 2

Find the common multiples of 3 and 4.

Solution:

Multiples of 3:

0, 3, 6, 9, 12, 15, 18, 21, 24...

Multiples of 4:

0, 4, 8, 12, 16, 20, 24, 28 ...

Common multiples of 3 and 4 are 0, 12, 24 and so on.

Co-prime Numbers

If 1 is the only common factor between two numbers then they are said to be **Co-prime Numbers**.

Example

Check whether 7 and 15 are co-prime numbers or not.

Solution:

Factors of 7 are 1 and 7.

Factors of 15 are 1, 3, 5 and 15.

The common factor of 7 and 15 is 1 only. Hence, they are the co-prime numbers.

Some more Divisibility Rules

1. Let a and b are two given numbers. If a is divisible by b then it will be divisible by all the factors of b also.

If 24 is divisible by 12 then 24 will be divisible by all the factors of 12(i.e.2, 3, 4, 6) also.

2. Let a and b are two co-prime numbers. If c is divisible by a and b then c will be divisible by the product of a and b (ab) also.

If 24 is divisible by 2 and 3 which are the co-prime numbers then 24 will also be divisible by the product of 2 and 3 (2×3=6).

3. If a and b are divisible by c then a + b will also be divisible by c.

If 24 and 12 are divisible by 4 then 24 + 12 = 36 will also be divisible by 4.

4. If a and b are divisible by c then a-b will also be divisible by c.

If 24 and 12 are divisible by 4 then 24 -12 = 12 will also be divisible by 4.

EXERCISE 3.4

1. Find the common factors of :
   (a) 20 and 28 (b) 15 and 25 (c) 35 and 50 (d) 56 and 120
2. Find the common factors of :
   (a) 4, 8 and 12 (b) 5, 15 and 25
3. Find first three common multiples of :
   (a) 6 and 8 (b) 12 and 18
4. Write all the numbers less than 100 which are common multiples of 3 and 4.
5. Which of the following numbers are co-prime?
   (a) 18 and 35 (b) 15 and 37 (c) 30 and 415
   (d) 17 and 68 (e) 216 and 215 (f) 81 and 16
6. A number is divisible by both 5 and 12. By which other number will that number be always divisible?
7. A number is divisible by 12. By what other numbers will that number be divisible?

Prime Factorisation

Prime Factorisation is the process of finding all the prime factors of a number.

There are **two methods** to find the prime factors of a number-

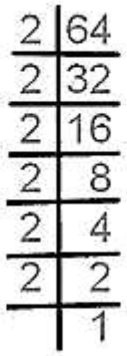
1. Prime factorisation using a factor tree

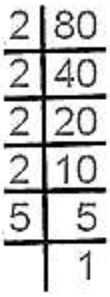
We can find the prime factors of 70 in two ways.

The prime factors of 70 are 2, 5 and 7 in both the cases.

2. Repeated Division Method

Find the prime factorisation of 64 and 80.





The prime factorisation of 64 is 2 × 2 × 2 × 2 × 2 × 2.

The prime factorisation of 80 is 2 × 2 × 2 × 2 × 5.

Highest Common Factor (HCF)

The highest common factor (HCF) of two or more given numbers is the greatest of their common factors.

Its other name is **(GCD) Greatest Common Divisor**.

Method to find HCF

To find the HCF of given numbers, we have to find the prime factorisation of each number and then find the HCF.

Example

Find the HCF of 60 and 72.

Solution:

First, we have to find the prime factorisation of 60 and 72.

Then encircle the common factors.

$$60 = 2 \times 2 \times 3 \times 5$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

HCF of 60 and 72 is 2 × 2 × 3 = 12.

Lowest Common Multiple (LCM)

The lowest common multiple of two or more given number is the smallest of their common multiples.

Methods to find LCM

1. Prime Factorisation Method

To find the LCM we have to find the prime factorisation of all the given numbers and then multiply all the prime factors which have occurred a maximum number of times.

Example

Find the LCM of 60 and 72.

Solution:

First, we have to find the prime factorisation of 60 and 72.

Then encircle the common factors.

$$60 = 2 \times 2 \times 3 \times 5$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

To find the LCM, we will count the common factors one time and multiply them with the other remaining factors.

LCM of 60 and 72 is $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

2. Repeated Division Method

If we have to find the LCM of so many numbers then we use this method.

Example

Find the LCM of 105, 216 and 314.

Solution:

Use the repeated division method on all the numbers together and divide until we get 1 in the last row.

2	105	216	314
2	105	108	157
2	105	54	157
3	105	27	157
3	35	9	157
3	35	3	157
5	35	1	157
7	7	1	157
157	1	1	157
	1	1	1

LCM of 105,216 and 314 is 2 × 2 × 2 × 3 × 3 × 3 × 5 × 7 × 157 = 1186920

Real life problems related to HCF and LCM

Example: 1

There are two containers having 240 litres and 1024 litres of petrol respectively. Calculate the maximum capacity of a container which can measure the petrol of both the containers when used an exact number of times.

Solution:

As we have to find the capacity of the container which is the exact divisor of the capacities of both the containers, i. e. **maximum capacity**, so we need to calculate the HCF.

2	240
2	120
2	60
2	30
3	15
5	5
	1

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

The common factors of 240 and 1024 are 2 × 2 × 2 × 2. Thus, the HCF of 240 and 1024 is 16. Therefore, the maximum capacity of the required container is 16 litres.

Example: 2

What could be the least number which when we divide by 20, 25 and 30 leaves a remainder of 6 in every case?

Solution:

As we have to find the least number so we will calculate the LCM first.

2	20	25	30
2	10	25	15
3	5	25	15
5	5	25	5
5	1	5	1
	1	1	1

So, LCM = $2 \times 2 \times 3 \times 5 \times 5$.

LCM of 20, 25 and 30 is 2 × 2 × 3 × 5 × 5 = 300.

Here 300 is the least number which when divided by 20, 25 and 30 then they will leave remainder 0 in each case. But we have to find the least number which leaves remainder 6 in all cases. Hence, the required number is 6 more than 300.

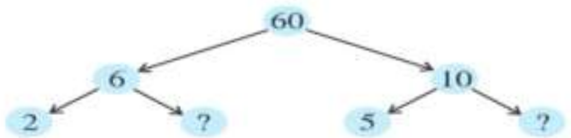
The required least number = 300 + 6 = 306.

EXERCISE 3.5

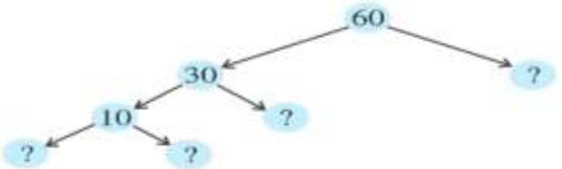
1. Which of the following statements are true?
   (a) If a number is divisible by 3, it must be divisible by 9.
   (b) If a number is divisible by 9, it must be divisible by 3.
   (c) A number is divisible by 18, if it is divisible by both 3 and 6.
   (d) If a number is divisible by 9 and 10 both, then it must be divisible by 90.
   (e) If two numbers are co-primes, at least one of them must be prime.
   (f) All numbers which are divisible by 4 must also be divisible by 8.
   (g) All numbers which are divisible by 8 must also be divisible by 4.
   (h) If a number exactly divides two numbers separately, it must exactly divide their sum.
   (i) If a number exactly divides the sum of two numbers, it must exactly divide the two numbers separately.

2. Here are two different factor trees for 60. Write the missing numbers.

(a)



(b)



3. Which factors are not included in the prime factorisation of a composite number?
4. Write the greatest 4-digit number and express it in terms of its prime factors.
5. Write the smallest 5-digit number and express it in the form of its prime factors.
6. Find all the prime factors of 1729 and arrange them in ascending order. Now state the relation, if any; between two consecutive prime factors.
7. The product of three consecutive numbers is always divisible by 6. Verify this statement with the help of some examples.
8. The sum of two consecutive odd numbers is divisible by 4. Verify this statement with the help of some examples.
9. In which of the following expressions, prime factorisation has been done?
   (a) $24 = 2 \times 3 \times 4$ (b) $56 = 7 \times 2 \times 2 \times 2$
   (c) $70 = 2 \times 5 \times 7$ (d) $54 = 2 \times 3 \times 9$
10. Determine if 25110 is divisible by 45.
    [**Hint :** 5 and 9 are co-prime numbers. Test the divisibility of the number by 5 and 9].
11. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify your answer.
12. I am the smallest number, having four different prime factors. Can you find me?

EXERCISE 3.6

1. Find the HCF of the following numbers :

   (a) 18, 48 (b) 30, 42 (c) 18, 60 (d) 27, 63

   (e) 36, 84 (f) 34, 102 (g) 70, 105, 175

   (h) 91, 112, 49 (i) 18, 54, 81 (j) 12, 45, 75

2. What is the HCF of two consecutive

   (a) numbers? (b) even numbers? (c) odd numbers?

EXERCISE 3.7

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.
2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?
3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.
4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.
5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.
6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?
7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.
8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.
9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.
10. Find the LCM of the following numbers :

    (a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

    Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?
11. Find the LCM of the following numbers in which one number is the factor of the other.

    (a) 5, 20 (b) 6, 18 (c) 12, 48 (d) 9, 45

    What do you observe in the results obtained?