ACADEMIC WORLD SCHOOL™
BEMETARA

SUMMER VACATION ASSIGNMENT

SESSION 2024-25
CLASS: XII
SUBJECTS : SCIENCE

General Instructions:

1. Write in a clear and legible handwriting.

2. Complete all the homework in a separate subject Summer Vacation Homework Notebook.

3. **DO NOT COPY AND PASTE FROM THE INTERNET**. (Assignment will be rejected)

4. In case of reference from the internet, you may:

   A. Read the content from the internet, if you wish and paraphrase (Rewrite in your own words)

   B. Mention the source of your information by providing the link from the internet for the verification by the teachers.

5. Marks awarded will be counted in the final scores at the end of the session.

6. The Summer Vacation HW will be submitted immediately upon arrival to school after Summer Vacation.

7. For any assignment related query do post your question on E-Mail Id of respective subject teacher. List of Subject Teacher's E-Mail ID attached.

Note for the Parents:

Parents are requested to guide his/her wards to complete their assignments honestly and submit by the due date.

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SUMMER VACATION ASSIGNMENT
SESSION 2024-25
CLASS: XII
SUBJECT : ENGLISH CORE (301)

General Instructions:

(i) Attempt all the questions as per the instructions.
(ii) Do not exceed the word limits.
(iii) Make a separate copy for English holiday homework.
(iv) This Homework carries 10 marks.

I. Reading Comprehension **[2 marks]**

Read any one **story** of your choice and interest and list the following :

   1. Setting   2. Characters   3. Mood   4. Conflict   5. Plot   6.Summary

All stories are free to stream on following website:

https://stories.audible.com/start-listen

II. Short Composition [Attempt any 3]

Advertisement/Notices/Posters/Invitations/Replies- **[3 marks]**

[format-1 content -1 expression- 1]

1. Your friend, Manish Tripathi has invited you to attend his wedding anniversary. You cannot attend it as you have a family get-together on the same day. Write a polite letter, in not more than 50 words, expressing your inability to attend the function and wish him all the happiness on this joyous occasion. You are Mita/ Mahendra Juneja of 25, Vasundra Colony, Patna

2. A poster-making competition has been announced by the school authorities during 'Health Awareness Week' to make the public aware of the wisdom of obeying simple measures people can take to slow the spread of the pandemic COVID-19 caused by the novel corona virus in your town. Draft a notice announcing about the competition creating your own details.

3. You are the cultural secretary of your school. Write a notice in about 50 words inviting the names of students who would like to participate in the variety programme that you are planning in aid of an old age home in your city. Items may be in the form of solo and group singing, mono acting, magic show, dance performance, etc. Trials for the most suitable participants will be held during the zero periods every day.

4. The Rotary Club of your locality is looking for Volunteers to prepare food packets in a community kitchen to be distributed among the needy during the nationwide lockdown in Raipur. Write an advertisement on behalf of the president of the club in the classified columns of the local newspaper giving necessary details in not more than 50 words.

III. Long Composition [Attempt any 5]
Letter writing - [Editorial, Placing Order, Complaint, Enquiry, Official, Institutional, Job Application] **[5 Marks]**
[Format-1, content-2, expression -2]

1. You are Natasha, residing in Pune. Your cousin, from the same city is hosting your grandmother's eightieth birth anniversary and has extended an invite to you. He has also requested your assistance for arrangements needed. Draft a reply of acceptance, in not more than 50 words.

   2. Your friend, Manish Tripathi has invited you to attend his wedding anniversary. You cannot attend it as you have a family get-together on the same day. Write a polite letter, in not more than 50 words, expressing your inability to attend the function and wish him all the happiness on this joyous occasion. You are Mita/ Mahendra Juneja of 25, Vasundra Colony, Patna

3. You are Satish/Sonali, the administrative officer of Apollo Hospital, New Delhi. You have been asked by the hospital management to place an order for masks, surgical gowns and eye gear including ventilators and personal protective equipment (PPE) for the medical staff. Write a letter to GPC Medical Ltd, Street No: 5; Kabir Nagar, Delhi, 110094 placing an order for the equipment. Invent the necessary details.

4. Public health care in the country is poor. Private health care is expensive. About a quarter of the population is illiterate and thus may not be as aware of the danger. And a massive citizenry makes social distancing nearly impossible — a big issue when around 100 million people in the country are over the age of 60. Write a letter to the Health
   Minister of India to provide comprehensive health coverage for all and to focus on health programmes to provide good health care to every citizen of India.

5. You are Nitin/ Natasha, staying at 20, S.F.S. Flats, Worli, Mumbai There are reports that the Owners of Fair Price Shops were found to be indulging in black marketing. Write a letter to the Commissioner of the Civil Supplies department, Mumbai to take strict actions against such shop owners as it is causing a lot of inconvenience to the people around your locality.

6. Write an application (including a resume) in 120-150 words for the post of receptionist advertised in a national newspaper by JKL Publishers, Peshwa Road, Pune. You are Karuna, M-114, Mall Road, Pune, a graduate from SNDT University and have done a Secretarial Practice Course from YWCA, Mumbai.
   Note: **The Summer holiday assignment is to be prepared in a Project File and submission is mandatory on the reopening of the school after vacation.**

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SUMMER VACATION ASSIGNMENT
SESSION 2024-25
CLASS: XII
SUBJECTS : MATHEMATICS

TO BE DONE IN LAB MANUAL

ACTIVITY 1 : To verify that the relation R in the set L of all lines in a plane, defined by R = {(l, m) : l ⊥ m} is symmetric but neither reflexive nor transitive.

ACTIVITY 2 : To demonstrate a function which is not one-one but is onto.

ACTIVITY 3 : To draw the graph of sin x $^{-1}$, using the graph of sin x and demonstrate the concept of mirror reflection (about the line y = x).

ACTIVITY 4 : To find analytically the limit of a function f (x) at x = c and also to check the continuity of the function at that point.

ACTIVITY 5 : To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base e) of the number x.

ACTIVITY 6 : To understand the concepts of local maxima, local minima and point of inflection.

TO BE DONE IN MILKY FILE

QUESTION : Write a brief description on any one topic given below (page limit at least 10)

1. RELATION
2. MATRICS
3. APPLICATION OF DERIVATIVE
4. PROBABILITY
5. APPLICATION OF INTEGRATION
6. LINEAR PROGRAMMING

ACADEMIC WORLD SCHOOL™
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SUMMER VACATION ASSIGNMENT

SESSION 2024-25

CLASS: XII

SUBJECT : BIOLOGY

Note-
1. Section A contains Investigatory Project
2. Section B contains EXPERIMENTS
3. Make Investigatory project in a Proper File
4. Write worksheet question answer either in fair note book or in assignment copy

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SECTION- A (Investigatory Project)

1. Make an investigatory project based on any one topic of your choice .The following points are to be taken care while preparing project-
2. Relevant topic must be chosen from the text book
3. Project must be handwritten.
4. Proper evidences (Data, pictures etc.) are to be produced in favour.
5. Project should not be copied from any source rather put your own effort.
6. Use internet for more information.
7. **You may choose other relevant topics of your choice other than suggested.**

Suggested topics
1. Any human disease can be taken as a topic and collect the complete information with statistical data and a support of statistical analysis about the same questioners.
2. Study about sleep walking and sleep paralysis.
3. Cancer.
4. Genetic and chromosomal disorders.
5. Study of locomotion in fishes, importance of different fins in balancing and steering the body. (M.R.- a fish tank, live fishes, scissors, petridishes, cotton).
6. Effects on plant movement (effects of light and effects of gravity). (M.R.- a potted plant, maize grains / bean / green gram seeds, petridishes, cotton).
7. Medicinal plants and their benefits.
8. Environmental Pollution
9. Infertility and steps that can be taken to overcome infertility problems.
10. To study the variation in the rate of mitotic cell division in the root tips of onion.
11. Effect of Plant growth regulators in development of plants
12. Effect of salinity of water on the growth of one type of plant.

13. Conduct a survey of pesticides at your local nursery, garden supplies shop or supermarket. Construct a table in which to record:
   a. the names of commercial brands of insecticides
   b. the target organisms
   c. the active chemical ingredients
   d. information given about safety precautions.
14. Stages of fetal development.
15. Find out how ants follow a trail, and how and why birds migrate.
16. Find out some of the innate behaviors of babies. Why might they be useful to a baby?
17. Investigate the statement "Too much adrenaline can cause stress-related diseases".
18. Design an experiment to compare the pH of various brands of toothpaste. What does the pH of toothpaste suggest about tooth decay?
19. What causes pimples? Why are they so difficult to prevent or cure?
20. Tobacco and its side effect.
21. STDs(symptoms, prevention & cure)

SECTION – B (PRACTICAL)

(A) Given below are the list of experiments. Write all the experiments in your Lab Manual as per the guidance of your subject teacher

A. List of Experiments

1. Prepare a temporary mount to observe pollen germination.

2. Study the plant population density by quadrat method.

3. Study the plant population frequency by quadrat method.

4. Prepare a temporary mount of onion root tip to study mitosis.

5. Isolate DNA from available plant material such as spinach, green pea seeds, papaya, etc.

ACADEMIC WORLD SCHOOL™
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SUMMER VACATION ASSIGNMENT

SESSION 2024-25

CLASS: XII (PCM/PCB)

SUBJECT : PHYSICS

1. **Make an investigatory project file on any ONE of the topics mentioned below. The file should be well designed and hand written .The following elements should be present in the file :**
   a) Cover page
   b) Certificate
   c) Index
   d) Aim
   e) Apparatus required
   f) Circuit diagram / Ray diagram
   g) Principle
   h) Procedure
   i) Observation table
   j) Result
   k) Precautions
   l) Bibliography

1. To study various factors on which the internal resistance/EMF of a cell depends.
2. To study the variations in current flowing in a circuit containing an LDR because of a variation in (a) the power of the incandescent lamp, used to 'illuminate' the LDR (keeping all the lamps at a fixed distance). (b) the distance of a incandescent lamp (of fixed power) used to 'illuminate' the LDR.

3. To find the refractive indices of (a) water (b) oil (transparent) using a plane mirror, an equiconvex lens (made from a glass of known refractive index) and an adjustable object needle.

4. To investigate the relation between the ratio of (i) output and input voltage and (ii) number of turns in the secondary coil and primary coil of a self-designed transformer.

5. To investigate the dependence of the angle of deviation on the angle of incidence using a hollow prism filled one by one, with different transparent fluids.

6. To estimate the charge induced on each one of the two identical Styrofoam (or pith) balls suspended in a vertical plane by making use of Coulomb's law.

7. To study the factor on which the self-inductance of a coil depends by observing the effect of this coil, when put in series with a resistor/(bulb) in a circuit fed up by an A.C. source of adjustable frequency.

8. To study the earth's magnetic field using a compass needle -bar magnet by plotting magnetic field lines and tangent galvanometer.

2. Find CBSE board questions from 2015 to 2022 of following chapters and solve it .

1. Ray optics.
2. Wave optics

3. Solve all exercise and in-text questions of Chapter 9 and Chapter 10 from NCERT.

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SUMMER VACATION ASSIGNMENT
SESSION 2024-25
CLASS: XII
SUBJECT : CHEMISTRY

A. **Make a investigatory project on any one of the following topic:**

1) On green chemistry- Bio-diesel and bio-petrol.

2) On sterilization of water using bleaching powder

3) On analysis of fertilizers

4) On measuring the amount of Acetic acid in vinegar

5) On determination of contents of cold drinks

6) To study the quantity of casein in milk

7) On presence of insecticides and pesticides in fruits and vegetables

8) On preparation of soyabean milk

9) On study of rate of fermentation of juices

10) Determination of caffeine in tea samples

11) Determination of content of toothpaste

12) Dyeing of wool, silk and cotton in malachite green

13) Effect of Potassium Bisulphite as a food preservatives

14) Study the rates of fermentation of fruit or vegetable juices

15) To Study and Analyse the Foaming Capacity of the Soap

16) To Determine which Antacid could Neutralize the most Stomach Acid

17) To Study the Various Effects of Dye on Different Types of Fabric

18) To Study the Power of Enzymes and Biotechnological Applications

19) Does Water Temperature Cause Metal to Expand and Contract

20) To Investigate the Methods of Preparation of Toilet Soaps

Things should be mentioned:

a) Aim

b)   Apparatus required

c)   Chemicals used

d)   Principle

e)   Procedure

f)   Observation table

g)   Result

h)   Precautions

i)   Bibliography

B. Solve the following worksheet according to the marks given below.

CH-2: SOUTIONS

VSA QUESTIONS

1.   Define the following terms:
a)   Mole fraction
b)   Isotonic solutions
c)   ideal solution.
d)   Colligative properties
e)   molality
2.   Why it is better to find molality of a solution than its molarity?
3.   What is Normality . Explain what do you understand by 0.5N H2SO4.
4.   What is vapour pressure . How it is related with boiling of a solution .
5.   Define Henry's law and write two of its application.
6.   Define Azeotropes and explain how they are classified ?

SA (I) TYPE QUESTIONS

7.   A mixture of chlorobenzene and bromobenzene is a nearly an ideal solution but a mixture of chloroform and acetone is not Explain?
8.   Suggest the most important type of intermolecular attractive interaction in the following pairs.
       (i)   n-hexane and n-octane
       (ii)   and
       (iii) and water
       (iv)   methanol and acetone
       (v)   acetonitrile and acetone
9.   Based on solute-solvent interactions, arrange the following in order of increasing solubility in n-octane and   explain. Cyclohexane, KCl.
10.   The vapour pressure of solvent gets lowered, when a non- volatile solute is added

       to it. Why?

11. (a) Explain the following :
    (i) Henry's law about dissolution of a gas in a liquid
   (ii) Boiling point elevation constant for a solvent

SA (II) TYPE QUESTIONS

12. Explain the following
    (a) Hypertonic Solution
    (b) Hypotonic Solution
    (c) Maximum Boiling Azeotropes with Example
13. Derive expression for Raoult's law when the solute is non-volatile.
14. What is meant by positive deviations from Raoult's law? Give an example. What is the sign of $\Delta_{mix}H$ for positive deviation?
15. (i) Gas (A) is more soluble in water than Gas at the same temperature. Which one of the two gases will have the higher value of K_H (Henry's constant) and why?

NUMERICALS

16. Give reasons for the following:
    (i) Measurement of osmotic pressure method is preferred for the determination of molar masses of macromolecules such as proteins and polymers.
    (ii) Aquatic animals are more comfortable in cold water than in warm water.
    (iii) Elevation of boiling point of 1M KCl solution is nearly double than that of 1 M sugar solution.
17. A 10% solution (by mass) of sucrose in water has freezing point of 269.15 K. Calculate the freezing point of 10% glucose in water, if freezing point of pure water is 273.15 K.
    Given: (Molar mass of sucrose = 342 g mol^{-1} , Molar mass of glucose = 180 g mol^{-1})
18. Two liquids A and B boil at 145^{0}C and 190^{0}C respectively. Which of them has a higher vapour pressure at 80^{0}C?
19. (a) Why is the vapour pressure of a solution of glucose in water lower than that of water?
    (b) A 6.90 M solution of KOH in water contains 30% by mass of KOH. Calculate the density of the KOH solution? (molar mass of KOH = 56 g/mol)

Activities for
Class XII
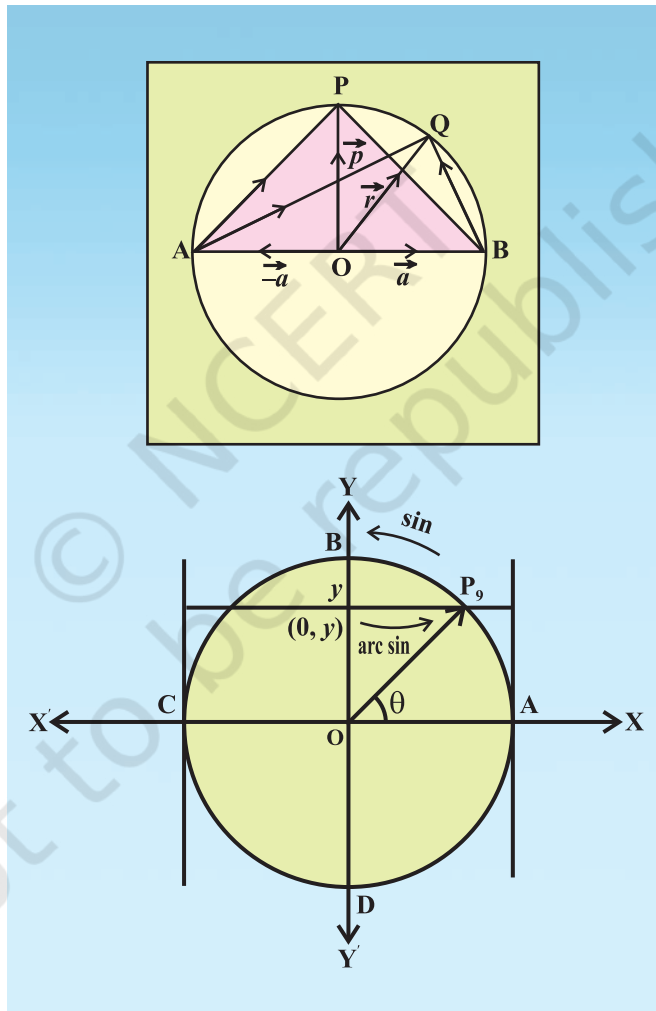
*The basic principles of learning mathematics are :
(a) learning should be related to each child individually
(b) the need for mathematics should develop from an
intimate acquaintance with the environment (c) the child
should be active and interested, (d) concrete material and
wide variety of illustrations are needed to aid the learning
process (e) understanding should be encouraged at each
stage of acquiring a particular skill (f) content should be
broadly based with adequate appreciation of the links
between the various branches of mathematics, (g) correct
mathematical usage should be encouraged at all stages.*

– Ronwill

Activity 1

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive.

MATERIAL REQUIRED

A piece of plywood, some pieces of wires (8), nails, white paper, glue etc.

METHOD OF CONSTRUCTION

Take a piece of plywood and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig.1.
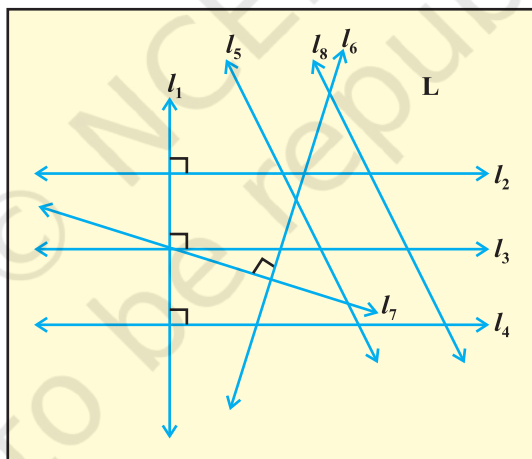
Fig. 1

DEMONSTRATION

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.

2. l_1 is perpendicular to each of the lines l_2, l_3, l_4. [see Fig. 1]

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in R$

OBSERVATION

1. In Fig. 1, no line is perpendicular to itself, so the relation R = {(l, m) : $l \perp m$} _____ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$? _____   (Yes/No)

   $\therefore$ (l_1, l_2) $\in$ R $\Rightarrow$ (l_2, l_1) _____ R ($\notin$ /$\in$)

   Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? _____   (Yes/No)

   $\therefore$ (l_3, l_1) $\in$ R $\Rightarrow$ (l_1, l_3) _____ R ($\notin$ /$\in$)

   Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? _____   (Yes/No)

   $\therefore$ (l_6, l_7) $\in$ R $\Rightarrow$ (l_7, l_6) _____ R ($\notin$ /$\in$)

   $\therefore$ The relation R .... symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? ... (Yes/No)

   i.e., $(l_2, l_1) \in$ R and $(l_1, l_3) \in$ R $\Rightarrow (l_2, l_3)$ _____ R ($\notin$ /$\in$)

   $\therefore$ The relation R .... transitive (is/is not).

APPLICATION

This activity can be used to check whether a given relation is an equivalence relation or not.

NOTE

1. In this case, the relation is not an equivalence relation.

2. The activity can be repeated by taking some more wire in different positions.

Activity 2

METHOD OF CONSTRUCTION

Take a piece of plywood of convenient size and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig. 2.
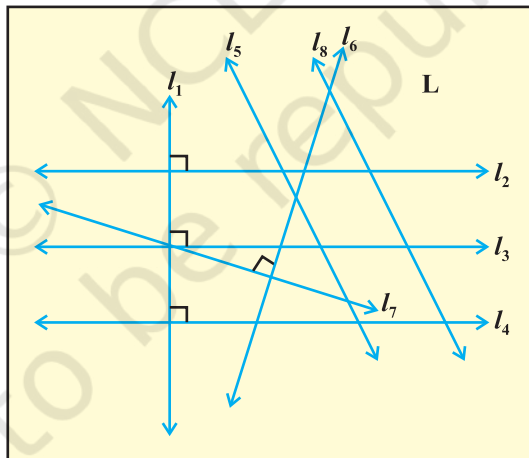

Fig. 2

DEMONSTRATION

1. Let the wires represent the lines l_1, l_2, ..., l_8.

2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (see Fig. 2).

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. (l_2, l_3), (l_3, l_4), (l_5, l_8), $\in$ R

OBSERVATION

1. In Fig. 2, every line is parallel to itself. So the relation R = {(l, m) : $l \parallel m$} .... reflexive relation (is/is not)

2. In Fig. 2, observe that $l_2 \parallel l_3$. Is $l_3 \dots l_2$? ($\cancel{\parallel}$/ $\parallel$)

   So, $(l_2, l_3) \in$ R $\Rightarrow (l_3, l_2) \dots$ R ($\notin$ /$\in$)
   Similarly, $l_3 \parallel l_4$. Is $l_4 \dots l_3$? ($\cancel{\parallel}$/ $\parallel$)
   So, $(l_3, l_4) \in$ R $\Rightarrow (l_4, l_3) \dots$ R ($\notin$ /$\in$)

   and $(l_5, l_8) \in$ R $\Rightarrow (l_8, l_5) \dots$ R ($\notin$ /$\in$)

$\therefore$ The relation R ... symmetric relation (is/is not)

3. In Fig. 2, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is $l_2 \dots l_4$? ($\parallel$ / $\parallel$)

   So, $(l_2, l_3) \in$ R and $(l_3, l_4) \in$ R $\Rightarrow (l_2, l_4) \dots$ R ($\in$ /$\notin$)

   Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is $l_3 \dots l_2$? ($\cancel{\parallel}$/ $\parallel$)
   So, $(l_3, l_4) \in$ R, $(l_4, l_2) \in$ R $\Rightarrow (l_3, l_2) \dots$ R ($\in$,$\notin$)

Thus, the relation R ... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

APPLICATION

This activity is useful in understanding the concept of an equivalence relation.

NOTE

This activity can be repeated by taking some more wires in different positions.

Activity 3

OBJECTIVE

To demonstrate a function which is not one-one but is onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails on it as shown in the Fig.3.1. Name the nails on the strip as 1, 2 and 3.

2. Paste another strip on the right hand side of the cardboard and fix two nails in the plastic strip as shown in Fig.3.2. Name the nails on the strip as *a* and *b*.

3. Join nails on the left strip to the nails on the right strip as shown in Fig. 3.3.

X

• 1

• 2

• 3

Fig. 3.1

Y

a •

b •

Fig. 3.2

1 •

2 •

3 •

• *a*

• *b*
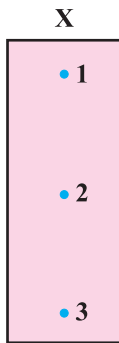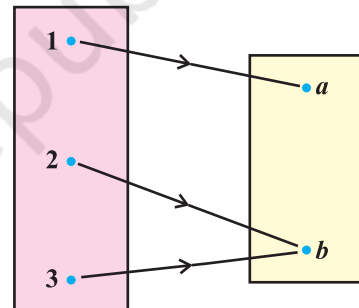
Fig. 3.3

DEMONSTRATION

1. Take the set X = {1, 2, 3}

2. Take the set Y = {*a*, *b*}

3. Join (correspondence) elements of X to the elements of Y as shown in Fig. 3.3

OBSERVATION

1. The image of the element 1 of X in Y is _____.

   The image of the element 2 of X in Y is _____.

The image of the element 3 of X in Y is _____.

So, Fig. 3.3 represents a _____ .

2. Every element in X has a _____ image in Y. So, the function is _____(one-one/not one-one).

3. The pre-image of each element of Y in X _____ (exists/does not exist). So, the function is _____ (onto/not onto).

APPLICATION

This activity can be used to demonstrate the concept of one-one and onto function.

Activity 4

OBJECTIVE

To demonstrate a function which is one-one but not onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix two nails in it as shown in the Fig. 4.1. Name the nails as a and b.

2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in the Fig. 4.2. Name the nails on the right strip as 1, 2 and 3.

3. Join nails on the left strip to the nails on the right strip as shown in the Fig. 4.3.
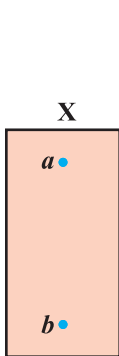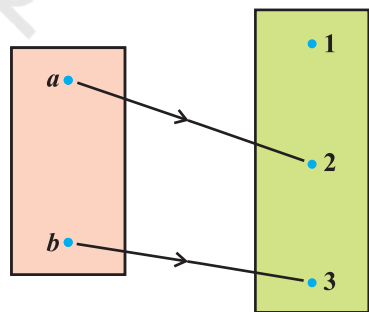
Fig. 4.1   Fig. 4.2   Fig. 4.3

DEMONSTRATION

1. Take the set X = {a, b}

2. Take the set Y = {1, 2, 3}.

3. Join elements of X to the elements of Y as shown in Fig. 4.3.

OBSERVATION

1. The image of the element *a* of X in Y is _____.

   The image of the element *b* of X in Y is _____.

   So, the Fig. 4.3 represents a _____.

2. Every element in X has a _____ image in Y. So, the function is _____ (one-one/not one-one).

3. The pre-image of the element 1 of Y in X _____ (exists/does not exist). So, the function is _____ (onto/not onto).

   Thus, Fig. 4.3 represents a function which is _____ but not onto.

APPLICATION

This activity can be used to demonstrate the concept of one-one but not onto function.

Activity 5

OBJECTIVE

To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$).

MATERIAL REQUIRED

Cardboard, white chart paper, ruler, coloured pens, adhesive, pencil, eraser, cutter, nails and thin wires.

METHOD OF CONSTRUCTION

1. Take a cardboard of suitable dimensions, say, 30 cm × 30 cm.

2. On the cardboard, paste a white chart paper of size 25 cm × 25 cm (say).

3. On the paper, draw two lines, perpendicular to each other and name them X′OX and YOY′ as rectangular axes [see Fig. 5].
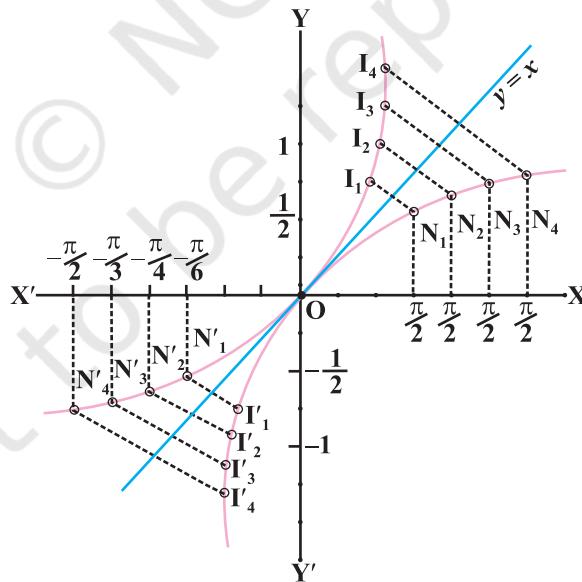

Fig. 5

4. Graduate the axes approximately as shown in Fig. 5.1 by taking unit on X-axis = 1.25 times the unit of Y-axis.

5. Mark approximately the points

$$\left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right), \left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right), \ldots, \left(\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$$ in the coordinate plane and at each point fix a nail.

6. Repeat the above process on the other side of the x-axis, marking the points

$$\left(\frac{-\pi}{6}, \sin\frac{-\pi}{6}\right), \left(\frac{-\pi}{4}, \sin\frac{-\pi}{4}\right), \ldots, \left(\frac{-\pi}{2}, \sin\frac{-\pi}{2}\right)$$ approximately and fix nails on these points as N_1', N_2', N_3', N_4'. Also fix a nail at O.

7. Join the nails with the help of a tight wire on both sides of x-axis to get the graph of sin x from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.

8. Draw the graph of the line $y = x$ (by plotting the points (1,1), (2, 2), (3, 3), ... etc. and fixing a wire on these points).

9. From the nails N_1, N_2, N_3, N_4, draw perpendicular on the line $y = x$ and produce these lines such that length of perpendicular on both sides of the line $y = x$ are equal. At these points fix nails, I_1, I_2, I_3, I_4.

10. Repeat the above activity on the other side of X- axis and fix nails at I_1', I_2', I_3', I_4'.

11. Join the nails on both sides of the line $y = x$ by a tight wire that will show the graph of $y = \sin^{-1} x$.

DEMONSTRATION

Put a mirror on the line $y = x$. The image of the graph of sin x in the mirror will represent the graph of $\sin^{-1} x$ showing that $\sin^{-1} x$ is mirror reflection of sin x and vice versa.

OBSERVATION

The image of point N_1 in the mirror (the line $y = x$) is _____.

The image of point N_2 in the mirror (the line $y = x$) is _____.

The image of point N_3 in the mirror (the line $y = x$) is _____.

The image of point N_4 in the mirror (the line $y = x$) is _____.

The image of point N_1' in the mirror (the line $y = x$) is _____.

The image point of N_2' in the mirror (the line $y = x$) is _____.

The image point of N_3' in the mirror (the line $y = x$) is _____.

The image point of N_4' in the mirror (the line $y = x$) is _____.

The image of the graph of six x in $y = x$ is the graph of _____, and the image of the graph of $\sin^{-1}x$ in $y = x$ is the graph of _____.

APPLICATION

Similar activity can be performed for drawing the graphs of $\cos^{-1}x$, $\tan^{-1}x$, etc.

Activity 6

OBJECTIVE

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

MATERIAL REQUIRED

Cardboard, white chart paper, rails, ruler, adhesive, steel wires and needle.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white chart paper on it.

2. Draw a unit circle with centre O on it.

3. Through the centre of the circle, draw two perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively as shown in Fig. 6.1.

4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig. 6.1.

5. Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.
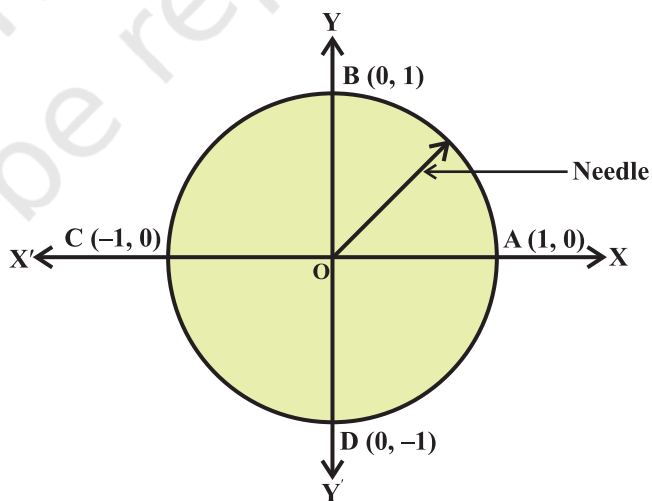
Fig. 6.1

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig. 6.2.
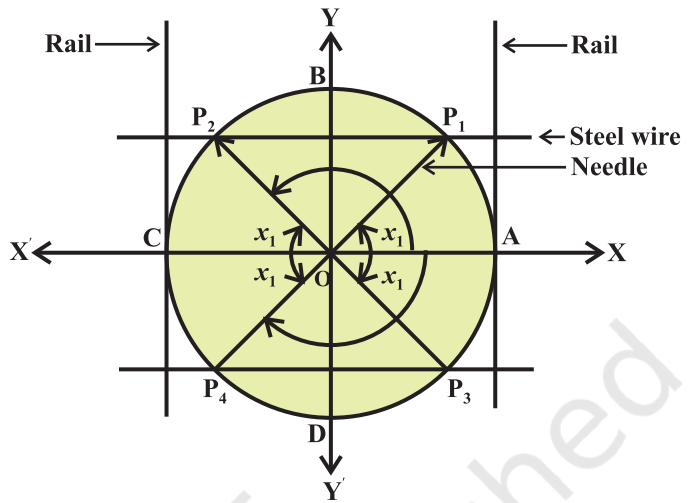

Fig. 6.2

DEMONSTRATION

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal to the length of intercepted arc of the unit circle.

2. Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needle (say P_1) (Fig. 6.2).

3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is the perpendicular distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$.

4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Find the value of y-coordinate of intersecting point P_2 with the help of sliding steel wire. Value of y-coordinate for the points P_1 and P_2 are same for the different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin (\pi - x_1)$. This demonstrates that sine function is not one-to-one for angles considered in first and second quadrants.

5. Keep the needle at angles $-x_1$ and $(-\pi + x_1)$, respectively. By sliding down the steel wire parallel to x-axis, demonstrate that y-coordinate for the points P_3 and P_4 are the same and thus sine function is not one-to-one for points considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. However, the *y*-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$ and look at the behaviour of *y*-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to *x*-axis accordingly. *y*-coordinate of points P_5, P_6, P_7 and P_8 are different (see Fig. 6.3). Hence, sine function is one-to-one in

Fig. 6.3

the domian $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$ and its range lies between -1 and 1.
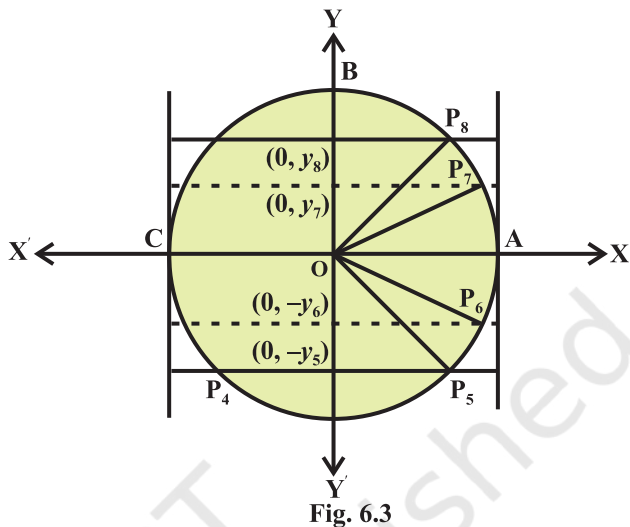
7. Keep the needle at any arbitrary angle say θ lying in the interval $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$

and denote the *y*-coordinate of the intersecting point P_9 as *y*. (see Fig. 6.4). Then $y = \sin \theta$ or $\theta = $ arc $\sin^{-1} y$) as sine function is one-one and onto in the

domain $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$ and range $[-1, 1]$. So, its inverse arc sine function exist. The domain of arc sine function is $[-1, 1]$ and
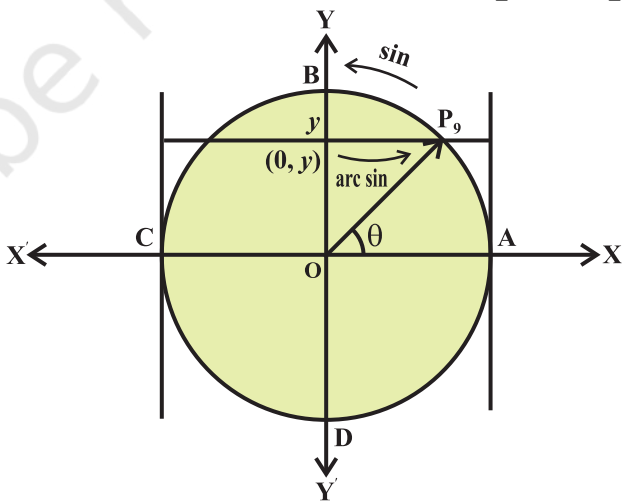
Fig. 6.4

range is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$. This range is called the principal value of arc sine function (or $\sin^{-1}$ function).

OBSERVATION

1. sine function is non-negative in _____ and _____ quadrants.

2. For the quadrants 3rd and 4th, sine function is _____.

3. $\theta = \text{arc sin } y \Rightarrow y = $ _____ θ where $-\dfrac{\pi}{2} \leq \theta \leq$ _____.

4. The other domains of sine function on which it is one-one and onto provides _____ for arc sine function.

APPLICATION

This activity can be used for finding the principal value of arc cosine function ($\cos^{-1}y$).

Activity 7

OBJECTIVE

To sketch the graphs of a^x and $\log_a x$, $a > 0$, $a \neq 1$ and to examine that they are mirror images of each other.

MATERIAL REQUIRED

Drawing board, geometrical instruments, drawing pins, thin wires, sketch pens, thick white paper, adhesive, pencil, eraser, a plane mirror, squared paper.

METHOD OF CONSTRUCTION

1. On the drawing board, fix a thick paper sheet of convenient size 20 cm × 20 cm (say) with adhesive.

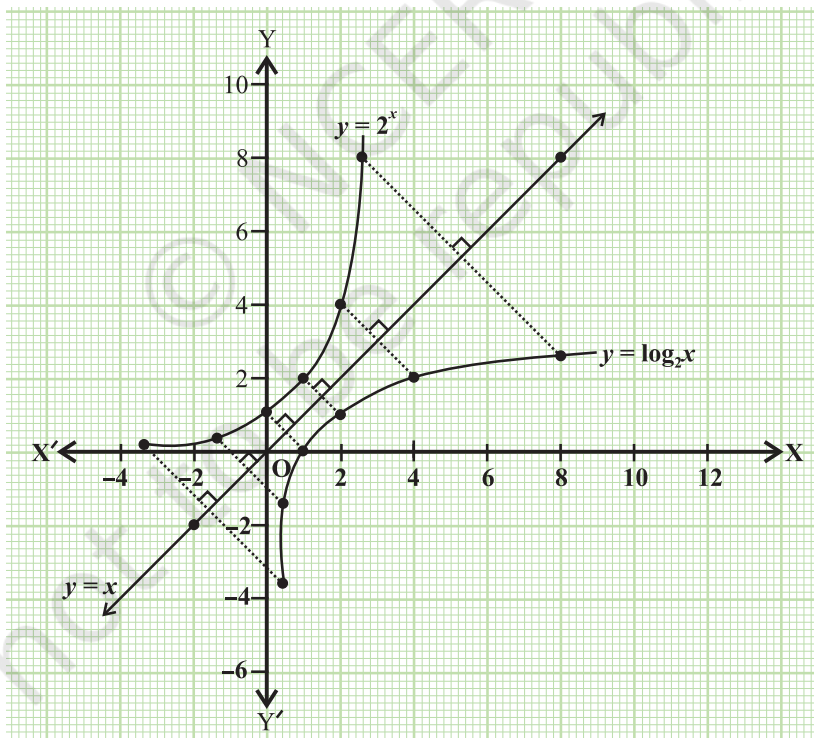**Fig. 7**

2. On the sheet, take two perpendicular lines XOX′ and YOY′, depicting coordinate axes.

3. Mark graduations on the two axes as shown in the Fig. 7.

4. Find some ordered pairs satisfying $y = a^x$ and $y = \log_a x$. Plot these points corresponding to the ordered pairs and join them by free hand curves in both the cases. Fix thin wires along these curves using drawing pins.

5. Draw the graph of $y = x$, and fix a wire along the graph, using drawing pins.

DEMONSTRATION

1. For a^x, take $a = 2$ (say), and find ordered pairs satisfying it as

x	0	1	−1	2	−2	3	−3	$\dfrac{1}{2}$	$-\dfrac{1}{2}$	4
2^x	1	2	0.5	4	$\dfrac{1}{4}$	8	$\dfrac{1}{8}$	1.4	0.7	16

and plot these ordered pairs on the squared paper and fix a drawing pin at each point.

2. Join the bases of drawing pins with a thin wire. This will represent the graph of 2^x.

3. $\log_2 x = y$ gives $x = 2^y$. Some ordered pairs satisfying it are:

x	1	2	$\dfrac{1}{2}$	4	$\dfrac{1}{4}$	8	$\dfrac{1}{8}$
y	0	1	−1	2	−2	3	−3

Plot these ordered pairs on the squared paper (graph paper) and fix a drawing pin at each plotted point. Join the bases of the drawing pins with a thin wire. This will represent the graph of $\log_2 x$.

4. Draw the graph of line $y = x$ on the sheet.

5. Place a mirror along the wire representing $y = x$. It can be seen that the two graphs of the given functions are mirror images of each other in the line $y = x$.

OBSERVATION

1. Image of ordered pair (1, 2) on the graph of $y = 2^x$ in $y = x$ is _____. It lies on the graph of $y = $ _____.

2. Image of the point (4, 2) on the graph $y = \log_2 x$ in $y = x$ is _____ which lies on the graph of $y = $ _____.

Repeat this process for some more points lying on the two graphs.

APPLICATION

This activity is useful in understanding the concept of (exponential and logarithmic functions) which are mirror images of each other in $y = x$.

Activity 8

OBJECTIVE

To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base e) of the number x.

MATERIAL REQUIRED

Hardboard, white sheet, graph paper, pencil, scale, log tables or calculator (graphic/scientific).

METHOD OF CONSTRUCTION

1. Paste a graph paper on a white sheet and fix the sheet on the hardboard.

2. Find some ordered pairs satisfying the function $y = \log_{10}x$. Using log tables/calculator and draw the graph of the function on the graph paper (see Fig. 8)
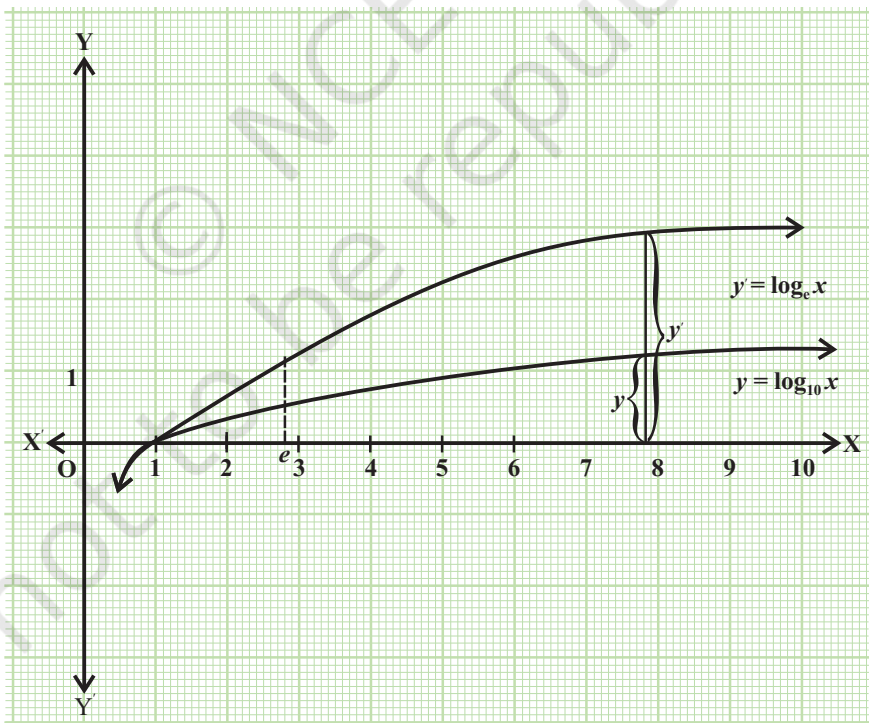

Fig. 8

3. Similarly, draw the graph of $y' = \log_e x$ on the same graph paper as shown in the figure (using log table/calculator).

DEMONSTRATION

1. Take any point on the positive direction of x-axis, and note its x-coordinate.

2. For this value of x, find the value of y-coordinates for both the graphs of $y = \log_{10} x$ and $y' = \log_e x$ by actual measurement, using a scale, and record them as y and y', respectively.

3. Find the ratio $\dfrac{y}{y'}$.

4. Repeat the above steps for some more points on the x-axis (with different values) and find the corresponding ratios of the ordinates as in Step 3.

5. Each of these ratios will nearly be the same and equal to 0.4, which is approximately equal to $\dfrac{1}{\log_e 10}$.

OBSERVATION

S.No.	Points on the x-axis	$y = \log_{10} x$	$y' = \log_e x$	Ratio $\dfrac{y}{y'}$ (approximate)
1.	$x_1 = $ _____	$y_1 = $ _____	$y'_1 = $ _____	_____
2.	$x_2 = $ _____	$y_2 = $ _____	$y'_2 = $ _____	_____
3.	$x_3 = $ _____	$y_3 = $ _____	$y'_3 = $ _____	_____
4.	$x_4 = $ _____	$y_4 = $ _____	$y'_4 = $ _____	_____
5.	$x_5 = $ _____	$y_5 = $ _____	$y'_5 = $ _____	_____
6.	$x_6 = $ _____	$y_6 = $ _____	$y'_6 = $ _____	_____

2. The value of $\dfrac{\overset{.}{y}}{y}$ for each point x is equal to _____ approximately.

3. The observed value of $\dfrac{\overset{.}{y}}{y}$ in each case is approximately equal to the value of $\dfrac{1}{\log_e 10}$. (Yes/No)

4. Therefore, $\log_{10} x = \dfrac{}{\log_e 10}$.

APPLICATION

This activity is useful in converting log of a number in one given base to log of that number in another base.

NOTE

Let, $y = \log_{10} x$, i.e., $x = 10^y$.

Taking logarithm to base e on both the sides, we get $\log_e x = y \log_e 10$

or $y = \dfrac{1}{\log_e 10}\left(\log_e x\right)$

$\Rightarrow \dfrac{\log_{10} x}{\log_e x} = \dfrac{1}{\log_e 10} = 0.434294$ (using log tables/calculator).

Activity 9

OBJECTIVE

To find analytically the limit of a function $f(x)$ at $x = c$ and also to check the continuity of the function at that point.

MATERIAL REQUIRED

Paper, pencil, calculator.

METHOD OF CONSTRUCTION

1. Consider the function given by
$$f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4}, & x \neq 4 \\ 10, & x = 4 \end{cases}$$

2. Take some points on the left and some points on the right side of c (= 4) which are very near to c.

3. Find the corresponding values of $f(x)$ for each of the points considered in step 2 above.

4. Record the values of points on the left and right side of c as x and the corresponding values of $f(x)$ in a form of a table.

DEMONSTRATION

1. The values of x and $f(x)$ are recorded as follows:

Table 1 : For points on the left of c (= 4).

x	3.9	3.99	3.999	3.9999	3.99999	3.999999	3.9999999
$f(x)$	7.9	7.99	7.999	7.9999	7.99999	7.999999	7.9999999

2. **Table 2**: For points on the right of c (= 4).

x	4.1	4.01	4.001	4.0001	4.00001	4.000001	4.0000001
$f(x)$	8.1	8.01	8.001	8.0001	8.00001	8.000001	8.0000001

OBSERVATION

1. The value of $f(x)$ is approaching to _____, as $x \to 4$ from the left.

2. The value of $f(x)$ is approaching to _____, as $x \to 4$ from the right.

3. So, $\lim\limits_{x \to 4} f(x)$ = _____ and $\lim\limits_{x \to 4^+} f(x)=$ _____.

4. Therefore, $\lim\limits_{x \to 4} f(x)=$ _____ , $f(4) =$ _____.

5. Is $\lim\limits_{x \to 4} f(x) = f(4)$ _____ ? (Yes/No)

6. Since $f(c) \neq \lim\limits_{x \to c} f(x)$, so, the function is _____ at $x = 4$ (continuous/ not continuous).

APPLICATION

This activity is useful in understanding the concept of limit and continuity of a function at a point.

Activity 10

OBJECTIVE

To verify that for a function f to be continuous at given point x_0,

$\Delta y = \left| f(x_0 + \Delta x) - f(x_0) \right|$ is

arbitrarily small provided. Δx is sufficiently small.

MATERIAL REQUIRED

Hardboard, white sheets, pencil, scale, calculator, adhesive.

METHOD OF CONSTRUCTION

1. Paste a white sheet on the hardboard.

2. Draw the curve of the given continuous function as represented in the Fig. 10.

3. Take any point A $(x_0, 0)$ on the positive side of x-axis and corresponding to this point, mark the point P (x_0, y_0) on the curve.
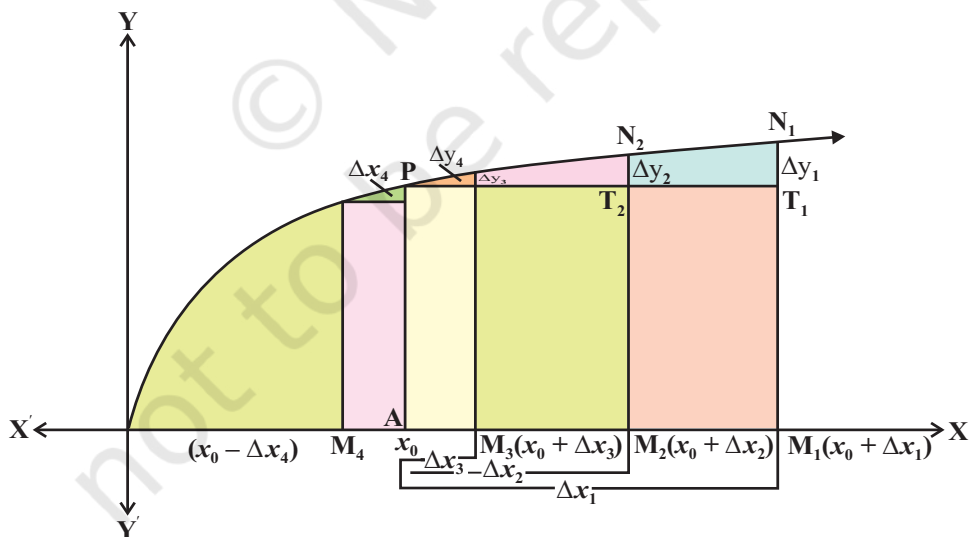

Fig. 10

DEMONSTRATION

1. Take one more point M_1 $(x_0 + \Delta x_1, 0)$ to the right of A, where Δx_1 is an increment in x.

2. Draw the perpendicular from M_1 to meet the curve at N_1. Let the coordinates of N_1 be $(x_0 + \Delta x_1, y_0 + \Delta y_1)$

3. Draw a perpendicular from the point P (x_0, y_0) to meet $N_1 M_1$ at T_1.

4. Now measure $AM_1 = \Delta x_1$ (say) and record it and also measure $N_1 T_1 = \Delta y_1$ and record it.

5. Reduce the increment in x to Δx_2 (i.e., $\Delta x_2 < \Delta x_1$) to get another point $M_2 \left(x_0 + \Delta x_2, 0 \right)$. Get the corresponding point N_2 on the curve

6. Let the perpendicular PT_1 intersects $N_2 M_2$ at T_2.

7. Again measure $AM_2 = \Delta x_2$ and record it.

   Measure $N_2 T_2 = \Delta y_2$ and record it.

8. Repeat the above steps for some more points so that Δx becomes smaller and smaller.

OBSERVATION

S.No.	Value of increment in x_0	Corresponding increment in y
1.	$\left|\Delta x_1\right| =$ ————	$\left|\Delta y_1\right| =$ ————
2.	$\left|\Delta x_2\right| =$ ————	$\left|\Delta y_2\right| =$ ————
3.	$\left|\Delta x_3\right| =$ ————	$\left|\Delta y_3\right| =$ ————
4.	$\left|\Delta x_4\right| =$ ————	$\left|\Delta y_4\right| =$ ————
5.	$\left|\Delta x_5\right| =$ ————	$\left|\Delta y_5\right| =$ ————

6.	$\left|\Delta x_6\right|=$	$\left|\Delta y_6\right|=$
7.	$\left|\Delta x_7\right|=$	$\left|\Delta y_7\right|=$
8.	$\left|\Delta x_8\right|=$	$\left|\Delta y_8\right|=$
9.	$\left|\Delta x_9\right|=$	$\left|\Delta y_9\right|=$

2. So, Δy becomes _____ when Δx becomes smaller.

3. Thus $\lim\limits_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

APPLICATION

This activity is helpful in explaining the concept of derivative (left hand or right hand) at any point on the curve corresponding to a function.

Activity 11

OBJECTIVE

To verify Rolle's Theorem.

MATERIAL REQUIRED

A piece of plywood, wires of different lengths, white paper, sketch pen.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.

2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent *x*-axis and *y*-axis (see Fig. 11).

3. Take a piece of wire of 15 cm length and bend it in the shape of a curve and fix it on the plywood as shown in the figure.
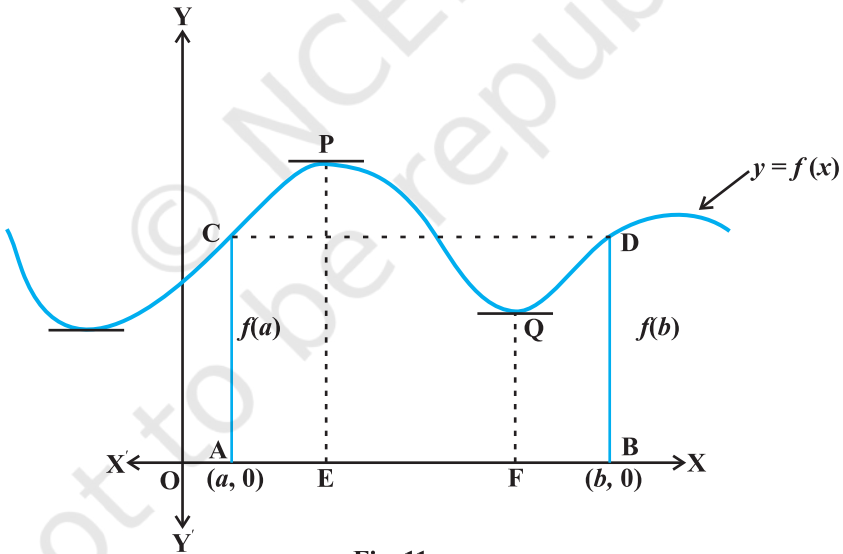
Fig. 11

4. Take two straight wires of the same length and fix them in such way that they are perpendicular to *x*-axis at the points A and B and meeting the curve at the points C and D (see Fig.11).

DEMONSTRATION

1. In the figure, let the curve represent the function $y = f(x)$. Let OA = a units and OB = b units.

2. The coordinates of the points A and B are $(a, 0)$ and $(b, 0)$, respectively.

3. There is no break in the curve in the interval $[a, b]$. So, the function f is continuous on $[a, b]$.

4. The curve is smooth between $x = a$ and $x = b$ which means that at each point, a tangent can be drawn which in turn gives that the function f is differentiable in the interval (a, b).

5. As the wires at A and B are of equal lengths, i.e., AC = BD, so $f(a) = f(b)$.

6. In view of steps (3), (4) and (5), conditions of Rolle's theorem are satisfied. From Fig.11, we observe that tangents at P as well as Q are parallel to x-axis, therefore, $f'(x)$ at P and also at Q are zero.

Thus, there exists at least one value c of x in (a,b) such that $f'(c) = 0$.

Hence, the Rolle's theorem is verified.

OBSERVATION

From Fig. 11.

$a = $ _____, $b = $ _____

$f(a) = $ _____, $f(b) = $ _____ Is $f(a) = f(b)$? (Yes/No)

Slope of tangent at P = _____, so, $f(x)$ (at P) =

APPLICATION

This theorem may be used to find the roots of an equation.

Activity 12

OBJECTIVE

To verify Lagrange's Mean Value
Theorem.

MATERIAL REQUIRED

A piece of plywood, wires, white
paper, sketch pens, wires.

METHOD OF CONSTRUCTION

1. Take a piece of plywood and paste a white paper on it.

2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent *x*-axis and *y*-axis (see Fig. 12).

3. Take a piece of wire of about 10 cm length and bend it in the shape of a curve as shown in the figure. Fix this curved wire on the white paper pasted on the plywood.
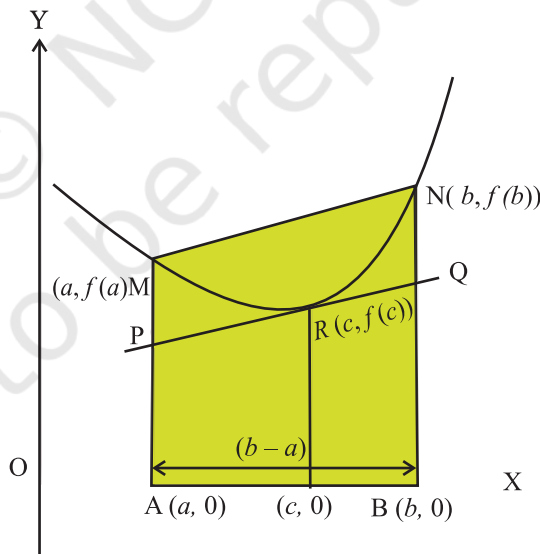
Fig. 12

4. Take two straight wires of lengths 10 cm and 13 cm and fix them at two different points of the curve parallel to y-axis and their feet touching the x-axis. Join the two points, where the two vertical wires meet the curve, using another wire.

5. Take one more wire of a suitable length and fix it in such a way that it is tangential to the curve and is parallel to the wire joining the two points on the curve (see Fig. 12).

DEMONSTRATION

1. Let the curve represent the function $y = f(x)$. In the figure, let OA $= a$ units and OB $= b$ units.

2. The coordinates of A and B are $(a, 0)$ and $(b, 0)$, respectively.

3. MN is a chord joining the points M $(a, f(a)$ and N $(b, f(b))$.

4. PQ represents a tangent to the curve at the point R $(c, f(c))$, in the interval (a, b).

5. $f'(c)$ is the slope of the tangent PQ at $x = c$.

6. $\dfrac{f(b) - f(a)}{b - a}$ is the slope of the chord MN.

7. MN is parallel to PQ, therefore, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. Thus, the Langrange's Mean Value Theorem is verified.

OBSERVATION

1. $a =$ _____, $b =$ _____,

   $f(a) =$ _____, $f(b) =$ _____.

2. $f(a) - f(b) =$ _____,

   $b - a =$ _____,

3. $\dfrac{f(b)-f(a)}{b-a}$ = _____ = Slope of MN.

4. Since PQ ∥ MN ⇒   Slope of PQ $= f'(c) = \dfrac{f(a)-f(a)}{b-a}$.

APPLICATION

Langrange's Mean Value Theorem has significant applications in calculus. For example this theorem is used to explain concavity of the graph.

Activity 13

OBJECTIVE

To understand the concepts of decreasing and increasing functions.

MATERIAL REQUIRED

Pieces of wire of different lengths, piece of plywood of suitable size, white paper, adhesive, geometry box, trigonometric tables.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires of length say 20 cm each and fix them on the white paper to represent x-axis and y-axis.
3. Take two more pieces of wire each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper as shown in the Fig. 13.
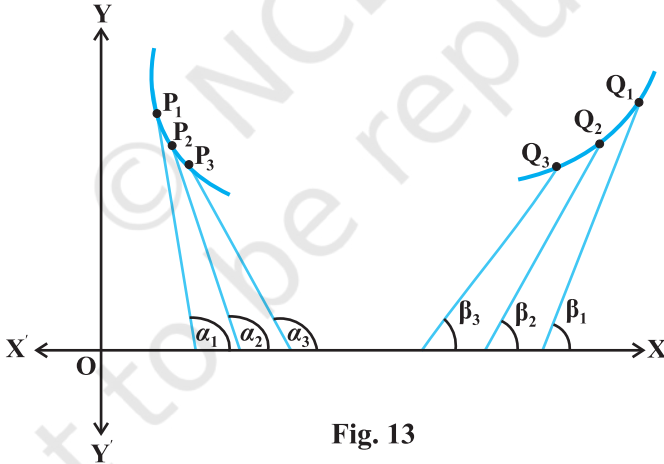

Fig. 13

4. Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.

DEMONSTRATION

1. Take one straight wire and place it on the curve (on the left) such that it is

tangent to the curve at the point say P_1 and making an angle α_1 with the positive direction of *x*-axis.

2. α_1 is an obtuse angle, so $\tan\alpha_1$ is negative, i.e., the slope of the tangent at P_1 (derivative of the function at P_1) is negative.

3. Take another two points say P_2 and P_3 on the same curve, and make tangents, using the same wire, at P_2 and P_3 making angles α_2 and α_3, respectively with the positive direction of *x*-axis.

4. Here again α_2 and α_3 are obtuse angles and therefore slopes of the tangents $\tan \alpha_2$ and $\tan \alpha_3$ are both negative, i.e., derivatives of the function at P_2 and P_3 are negative.

5. The function given by the curve (on the left) is a decreasing function.

6. On the curve (on the right), take three point Q_1, Q_2, Q_3, and using the other straight wires, form tangents at each of these points making angles β_1, β_2, β_3, respectively with the positive direction of *x*-axis, as shown in the figure. β_1, β_2, β_3 are all acute angles.

So, the derivatives of the function at these points are positive. Thus, the function given by this curve (on the right) is an increasing function.

OBSERVATION

1. $\alpha_1 = $ _____ , $> 90°$ $\alpha_2 = $ _____ $>$ _____, $\alpha_3 = $ _____$>$ _____, $\tan \alpha_1 = $ _____, (negative) $\tan \alpha_2 = $ _____, (_____), $\tan \alpha_3 = $ _____, (_____). Thus the function is _____.

2. $\beta_1 = $ _____$< 90°$, $\beta_2 = $ _____, $<$ _____, $\beta_3 = $ _____ , $<$ _____

$\tan \beta_1 = $ _____ , (positive), $\tan \beta_2 = $ _____, (_____), $\tan \beta_3 = $ _____(_____). Thus, the function is _____.

APPLICATION

This activity may be useful in explaining the concepts of decreasing and increasing functions.

Activity 14

OBJECTIVE

To understand the concepts of local maxima, local minima and point of inflection.

MATERIAL REQUIRED

A piece of plywood, wires, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.

2. Take two pieces of wires each of length 40 cm and fix them on the paper on plywood in the form of *x*-axis and *y*-axis.

3. Take another wire of suitable length and bend it in the shape of curve. Fix this curved wire on the white paper pasted on plywood, as shown in Fig. 14.
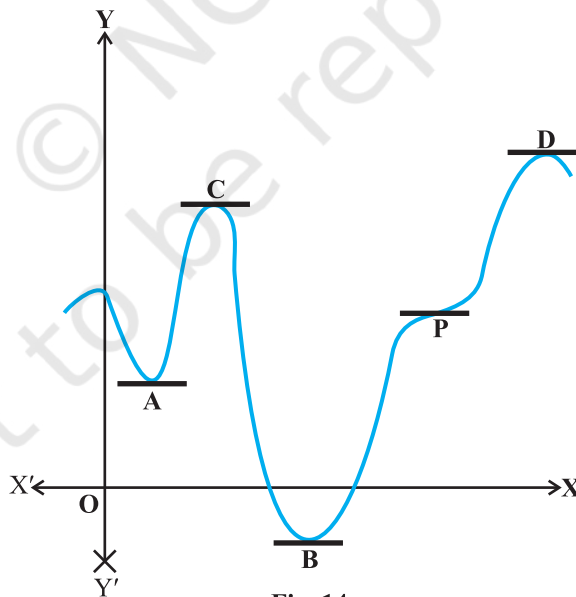

Fig. 14

4. Take five more wires each of length say 2 cm and fix them at the points A, C, B, P and D as shown in figure.

DEMONSTRATION

1. In the figure, wires at the points A, B, C and D represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e., the value of the first derivative at these points is zero. The tangent at P intersects the curve.

2. At the points A and B, sign of the first derivative changes from negative to positive. So, they are the points of local minima.

3. At the point C and D, sign of the first derivative changes from positive to negative. So, they are the points of local maxima.

4. At the point P, sign of first derivative does not change. So, it is a point of inflection.

OBSERVATION

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is _____.

2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is_____.

3. Sign of the first derivative at a point on the curve to immediate left of B is _____.

4. Sign of the first derivative at a point on the curve to immediate right of B is _____.

5. Sign of the first derivative at a point on the curve to immediate left of C is _____.

6. Sign of the first derivative at a point on the curve to immediate right of C is _____.

7. Sign of the first derivative at a point on the curve to immediate left of D is _____.

8. Sign of the first derivative at a point on the curve to immediate right of D is _____.

9. Sign of the first derivative at a point immediate left of P is _____ and immediate right of P is_____.

10. A and B are points of local _____.

11. C and D are points of local _____.

12. P is a point of _____.

APPLICATION

1. This activity may help in explaining the concepts of points of local maxima, local minima and inflection.

2. The concepts of maxima/minima are useful in problems of daily life such as making of packages of maximum capacity at minimum cost.

Activity 15

OBJECTIVE

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

MATERIAL REQUIRED

Drawing board, white chart paper, adhesive, geometry box, pencil and eraser, sketch pens, ruler, calculator.
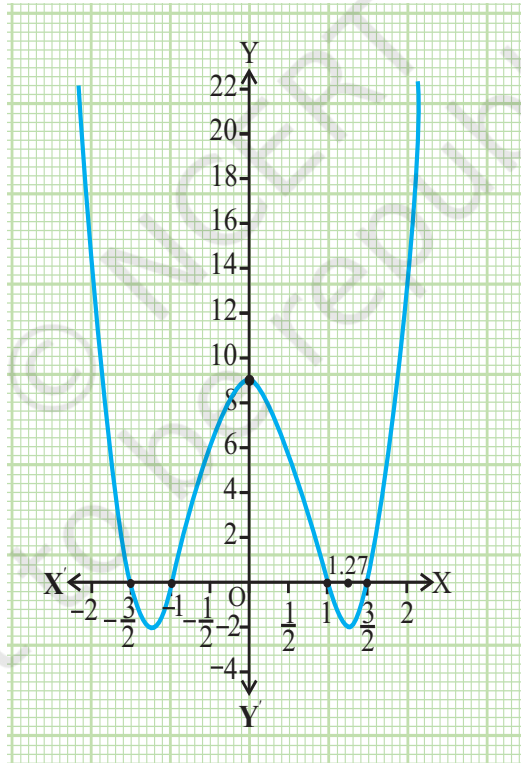

Fig 15

METHOD OF CONSTRUCTION

1. Fix a white chart paper of convenient size on a drawing board using adhesive.

2. Draw two perpendicular lines on the squared paper as the two rectangular axes.

3. Graduate the two axes as shown in Fig.15.

4. Let the given function be $f(x) = (4x^2 - 9)(x^2 - 1)$ in the interval $[-2, 2]$.

5. Taking different values of x in $[-2, 2]$, find the values of $f(x)$ and plot the ordered pairs $(x, f(x))$.

6. Obtain the graph of the function by joining the plotted points by a free hand curve as shown in the figure.

DEMONSTRATION

1. Some ordered pairs satisfying $f(x)$ are as follows:

x	0	± 0.5	± 1.0	1.25	1.27	± 1.5	± 2
$f(x)$	9	6	0	-1.55	-1.56	0	21

2. Plotting these points on the chart paper and joining the points by a free hand curve, the curve obtained is shown in the figure.

OBSERVATION

1. The absolute maximum value of $f(x)$ is _____ at $x =$ _____.

2. Absolute minimum value of $f(x)$ is _____ at $x =$ _____.

APPLICATION

The activity is useful in explaining the concepts of absolute maximum / minimum value of a function graphically.

Consider $f(x) = (4x^2 - 9)(x^2 - 1)$

$f(x) = 0$ gives the values of x as $\pm \dfrac{3}{2}$ and ± 1. Both these values of x lie in the given closed interval $[-2, 2]$.

$f'(x) = (4x^2 - 9) 2x + 8x(x^2 - 1) = 16x^3 - 26x = 2x(8x^2 - 13)$

$f'(x) = 0$ gives $x = 0$, $x = \pm \sqrt{\dfrac{13}{8}} = \pm 1.27$. These two values of x lie in $[-2, 2]$.

The function has local maxima/minima at $x = 0$ and $x = \pm 1.27$, respectively.

Activity 16

OBJECTIVE

To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

MATERIAL REQUIRED

Chart papers, scissors, cellotape, calculator.

METHOD OF CONSTRUCTION

1. Take a rectangular chart paper of size 20 cm × 10 cm and name it as ABCD.

2. Cut four equal squares each of side x cm from each corner A, B, C and D.

3. Repeat the process by taking the same size of chart papers and different values of x.

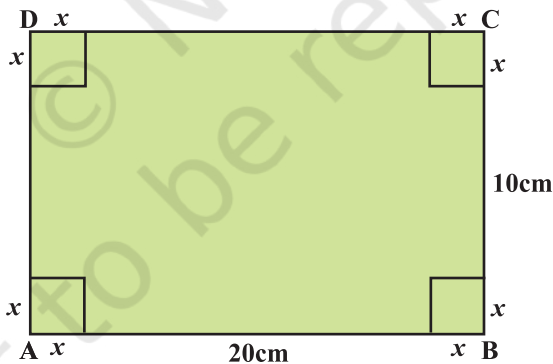4. Make an open box by folding its flaps using cellotape/adhesive.

Fig. 16

DEMONSTRATION

1. When $x = 1$, Volume of the box = 144 cm^3

2. When $x = 1.5$, Volume of the box = 178.5 cm^3

3. When $x = 1.8$, Volume of the box = 188.9 cm³.

4. When $x = 2$, Volume of the box = 192 cm³.

5. When $x = 2.1$, Volume of the box = 192.4 cm³.

6. When $x = 2.2$, Volume of the box = 192.2 cm³.

7. When $x = 2.5$, Volume of the box = 187.5 cm³.

8. When $x = 3$, Volume of the box = 168 cm³.

Clearly, volume of the box is maximum when $x = 2.1$.

OBSERVATION

1. V_1 = Volume of the open box (when $x = 1.6$) = ...

2. V_2 = Volume of the open box (when $x = 1.9$) = ...

3. V = Volume of the open box (when $x = 2.1$) = ...

4. V_3 = Volume of the open box (when $x = 2.2$) = ...

5. V_4 = Volume of the open box (when $x = 2.4$) = ...

6. V_5 = Volume of the open box (when $x = 3.2$) = ...

7. Volume V_1 is _____ than volume V.

8. Volume V_2 is _____ than volume V.

9. Volume V_3 is _____ than volume V.

10. Volume V_4 is _____ than volume V.

11. Volume V_5 is _____ than volume V.

So, Volume of the open box is maximum when $x = $ _____.

APPLICATION

This activity is useful in explaining the concepts of maxima/minima of functions. It is also useful in making packages of maximum volume with minimum cost.

Let V denote the volume of the box.

Now V = (20 − 2x) (10 − 2x) x

or V = $200x - 60x^2 + 4x^3$

$\dfrac{dV}{dx} = 200 - 120x + 12x^2$. For maxima or minima, we have,

$\dfrac{dV}{dx} = 0$, i.e., $3x^2 - 30x + 50 = 0$

i.e., $x = \dfrac{30 \pm \sqrt{900 - 600}}{6} = 7.9$ or 2.1

Reject x = 7.9.

$\dfrac{d^2V}{dx^2} = -120 + 24x$

When x = 2.1, $\dfrac{d^2V}{dx^2}$ is negative.

Hence, V should be maximum at x = 2.1.

Activity 17

OBJECTIVE

To find the time when the area of a rectangle of given dimensions become maximum, if the length is decreasing and the breadth is increasing at given rates.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser, cardboard.

METHOD OF CONSTRUCTION

1. Take a rectangle R_1 of dimensions 16 cm × 8 cm.

2. Let the length of the rectangle is decreasing at the rate of 1cm/second and the breadth is increasing at the rate of 2 cm/second.

3. Cut other rectangle R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, etc. of dimensions 15 cm × 10 cm, 14 cm × 12 cm, 13 cm × 14 cm, 12 cm × 16 cm, 11 cm × 18 cm, 10 cm × 20 cm, 9 cm × 22 cm, 8 cm × 24 cm (see Fig.17).

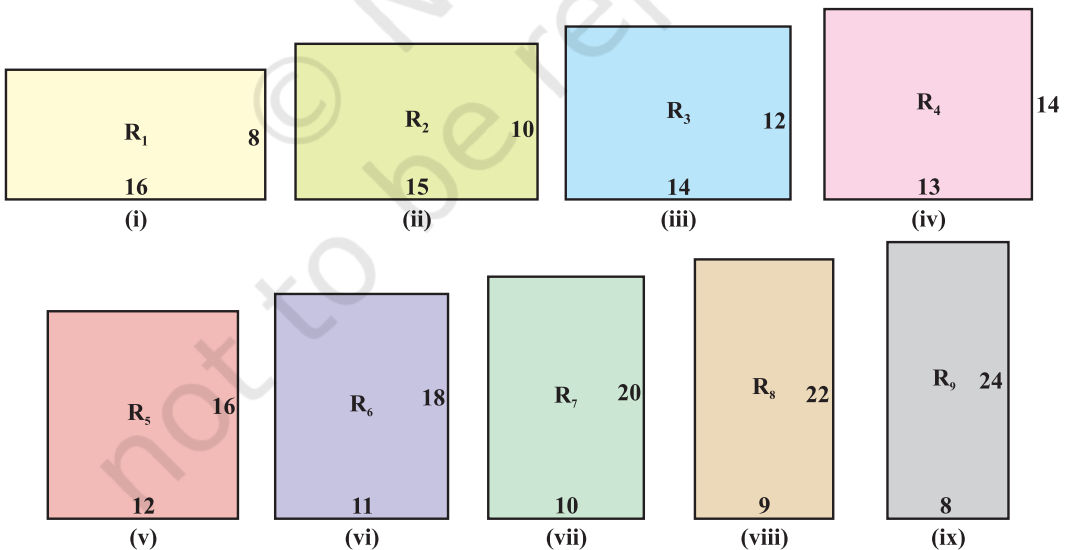4. Paste these rectangles on card board.

Fig. 17

DEMONSTRATION

1. Length of the rectangle is decreasing at the rate of 1cm/s and the breadth is increasing at the rate of 2cm/s.

2. (i)   Area of the given rectangle $R_1 = 16 \times 8 = 128$ cm^2.

   (ii)   Area of rectangle $R_2 = 15 \times 10 = 150$ cm^2 (after 1 sec).

   (iii) Area of rectangle $R_3 = 168$ cm^2 (after 2 sec).

   (iv)   Area of rectangle $R_4 = 182$ cm^2 (after 3 sec).

   (v)   Area of rectangle $R_5 = 192$ cm^2 (after 4 sec).

   (vi)   Area of rectangle $R_6 = 198$ cm^2 (after 5 sec).

   (vii) Area of rectangle $R_7 = 200$ cm^2 (after 6 sec).

   (viii) Area of rectangle $R_8 = 198$ cm^2 (after 7 sec) and so on.

Thus the area of the rectangle is maximum after 6 sec.

OBSERVATION

1. Area of the rectangle R_2 (after 1 sec) = _____.
2. Area of the rectangle R_4 (after 3 sec) = _____.
3. Area of the rectangle R_6 (after 5 sec) = _____.
4. Area of the rectangle R_7 (after 6 sec) = _____.
5. Area of the rectangle R_8 (after 7 sec) = _____.
6. Area of the rectangle R_9 (after 8 sec) = _____.
7. Rectangle of Maximum area (after   ..... seconds) = _____.
8. Area of the rectangle is maximum after _____ sec.
9. Maximum area of the rectangle is _____.

APPLICATION

This activity can be used in explaining the concept of rate of change and optimisation of a function.

The function has local maxima/minima at $x = 0$ and $x = \pm 1.27$, respectively.

Let the length and breadth of rectangle be a and b.

The length of rectangle after t seconds = $a - t$.

The breadth of rectangle after t seconds = $b + 2t$.

Area of the rectangle (after t sec) = $A(t) = (a - t)(b + 2t) = ab - bt + 2at - 2t^2$

$A'(t) = -b + 2a - 4t$

For maxima or minima, $A'(t) = 0$.

$A'(t) = 0 \Rightarrow t = \dfrac{2a - b}{4}$

$A''(t) = -4$

$A''\left(\dfrac{2a - b}{4}\right) = -4$, which is negative

Thus, $A(t)$ is maximum at $t = \dfrac{2a - b}{4}$ seconds.

Here, $a = 16$ cm, $b = 8$ cm.

Thus, $t = \dfrac{32 - 8}{4} = \dfrac{24}{4} = 6$ seconds

Hence, after 6 second, the area will become maximum.

Activity 18

OBJECTIVE

To verify that amongst all the rect-angles of the same perimeter, the square has the maximum area.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser cardboard, glue.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.

2. Make rectangles each of perimeter say 48 cm on a chart paper. Rectangles of different dimensions are as follows:
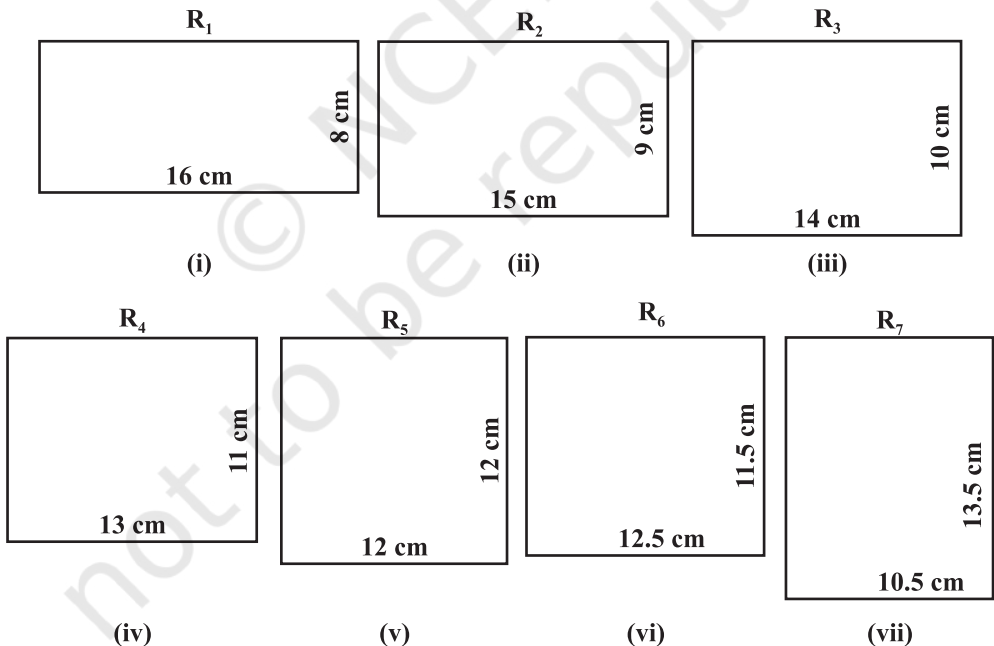
R₁ 16 cm × 8 cm (i)

R₂ 15 cm × 9 cm (ii)

R₃ 14 cm × 10 cm (iii)

R₄ 13 cm × 11 cm (iv)

R₅ 12 cm × 12 cm (v)

R₆ 12.5 cm × 11.5 cm (vi)

R₇ 10.5 cm × 13.5 cm (vii)

Fig. 18

R$_1$: 16 cm × 8 cm,   R$_2$: 15 cm × 9 cm

R$_3$: 14 cm × 10 cm,   R$_4$: 13 cm × 11 cm

R$_5$: 12 cm × 12 cm,   R$_6$: 12.5 cm × 11.5 cm

R$_7$: 10.5 cm × 13.5 cm

3. Cut out these rectangles and paste them on the white paper on the cardboard (see Fig. 18 (i) to (vii)).

4. Repeat step 2 for more rectangles of different dimensions each having perimeter 48 cm.

5. Paste these rectangles on cardboard.

DEMONSTRATION

1. Area of rectangle of R$_1$ = 16 cm × 8 cm = 128 cm^2

   Area of rectangle R$_2$ = 15 cm × 9 cm = 135 cm^2

   Area of R$_3$ = 140 cm^2

   Area of R$_4$ = 143 cm^2

   Area of R$_5$ = 144 cm^2

   Area of R$_6$ = 143.75 cm^2

   Area of R$_7$ = 141.75 cm^2

2. Perimeter of each rectangle is same but their area are different. Area of rectangle R$_5$ is the maximum. It is a square of side 12 cm. This can be verified using theoretical description given in the note.

OBSERVATION

1. Perimeter of each rectangle R$_1$, R$_2$, R$_3$, R$_4$, R$_4$, R$_6$, R$_7$ is _____.

2. Area of the rectangle R$_3$ _____ than the area of rectangle R$_5$.

3. Area of the rectangle R_6 _____ than the area of rectangle R_5.

4. The rectangle R_5 has the diamensions _____ × _____ and hence it is a _____.

5. Of all the rectangles with same perimeter, the _____ has the maximum area.

APPLICATION

This activity is useful in explaining the idea of Maximum of a function. The result is also useful in preparing economical packages.

NOTE

Let the length and breadth of rectangle be x and y.

The perimeter of the rectangle $P = 48$ cm.

$2(x + y) = 48$

or $x + y = 24$ or $y = 24 - x$

Let A (x) be the area of rectangle, then

A $(x) = xy$

$\quad = x(24 - x)$

$\quad = 24x - x^2$

A$'(x) = 24 - 2x$

A$'(x) = \Rightarrow 24 - 2x = 0 \Rightarrow x = 12$

A$''(x) = -2$

A$''(12) = -2$, which is negative

Therefore, area is maximum when $x = 12$

$y = x = 24 - 12 = 12$

So, $x = y = 12$

Hence, amongst all rectangles, the square has the maximum area.

Activity 19

OBJECTIVE

To evaluate the definite integral $\int_a^b \sqrt{(1-x^2)}\,dx$ as the limit of a sum and verify it by actual integration.

MATERIAL REQUIRED

Cardboard, white paper, scale, pencil, graph paper

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.

2. Draw two perpendicular lines to represent coordinate axes XOX′ and YOY′.

3. Draw a quadrant of a circle with O as centre and radius 1 unit (10 cm) as shown in Fig.19.

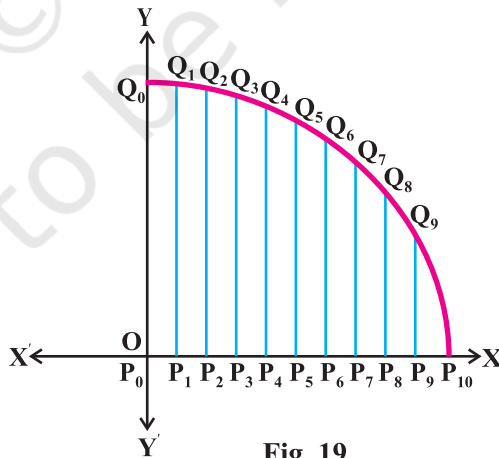The curve in the 1st quadrant represents the graph of the function $\sqrt{1-x^2}$ in the interval [0, 1].

Fig. 19

DEMONSTRATION

1. Let origin O be denoted by P_0 and the points where the curve meets the x-axis and y-axis be denoted by P_{10} and Q, respectively.

2. Divide P_0P_{10} into 10 equal parts with points of division as, $P_1, P_2, P_3, ..., P_9$.

3. From each of the points, P_i, $i = 1, 2, ..., 9$ draw perpendiculars on the x-axis to meet the curve at the points, $Q_1, Q_2, Q_3, ..., Q_9$. Measure the lengths of $P_0Q_0, P_1Q_1, ..., P_9Q_9$ and call them as $y_0, y_1, ..., y_9$ whereas width of each part, $P_0P_1, P_1P_2, ...,$ is 0.1 units.

4. $y_0 = P_0Q_0 = 1$ units

   $y_1 = P_1Q_1 = 0.99$ units

   $y_2 = P_2Q_2 = 0.97$ units

   $y_3 = P_3Q_3 = 0.95$ units

   $y_4 = P_4Q_4 = 0.92$ units

   $y_5 = P_5Q_5 = 0.87$ units

   $y_6 = P_6Q_6 = 0.8$ units

   $y_7 = P_7Q_7 = 0.71$ units

   $y_8 = P_8Q_8 = 0.6$ units

   $y_9 = P_9Q_9 = 0.43$ units

   $y_{10} = P_{10}Q_{10} =$ which is very small near to 0.

5. Area of the quadrant of the circle (area bounded by the curve and the two axis) = sum of the areas of trapeziums.

$$= \frac{1}{2} \times 0.1 \begin{bmatrix} (1+0.99) + (0.99+0.97) + (0.97+0.95) + (0.95+0.92) \\ + (0.92+0.87) + (0.87+0.8) + (0.8+0.71) + (0.71+0.6) \\ + (0.6+0.43) + (0.43) \end{bmatrix}$$

= 0.1 [0.5 + 0.99 + 0.97 + 0.95 + 0.92 + 0.87 + 0.80 + 0.71 + 0.60 + 0.43]

= 0.1 × 7.74 = 0.774 sq. units.(approx.)

6. Definite integral = $\int_0^1 \sqrt{1 - x^2}\, dx$

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x \right]_0^1 = \frac{1}{2} \times \frac{\pi}{2} = \frac{3.14}{4} = 0.785 \text{ sq.units}$$

Thus, the area of the quadrant as a limit of a sum is nearly the same as area obtained by actual integration.

OBSERVATION

1. Function representing the arc of the quadrant of the circle is $y =$ _____.

2. Area of the quadrant of a circle with radius 1 unit $= \int_0^1 \sqrt{1 - x^2}\, dx =$ _____.

   sq. units

3. Area of the quadrant as a limit of a sum = _____ sq. units.

4. The two areas are nearly _____.

APPLICATION

This activity can be used to demonstrate the concept of area bounded by a curve. This activity can also be applied to find the approximate value of π.

NOTE

Demonstrate the same activity by drawing the circle $x^2 + y^2 = 9$ and find the area between $x = 1$ and $x = 2$.

Activity 20

OBJECTIVE

To verify geometrically that

$$\vec{c}\times\left(\vec{a}+\vec{b}\right)=\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$$

MATERIAL REQUIRED

Geometry box, cardboard, white paper, cutter, sketch pen, cellotape.

METHOD OF CONSTRUCTION

1. Fix a white paper on the cardboard.

2. Draw a line segment OA (= 6 cm, say) and let it represent $\vec{c}$.

3. Draw another line segment OB (= 4 cm, say) at an angle (say 60°) with OA. Let $\overrightarrow{OB} = \vec{a}$
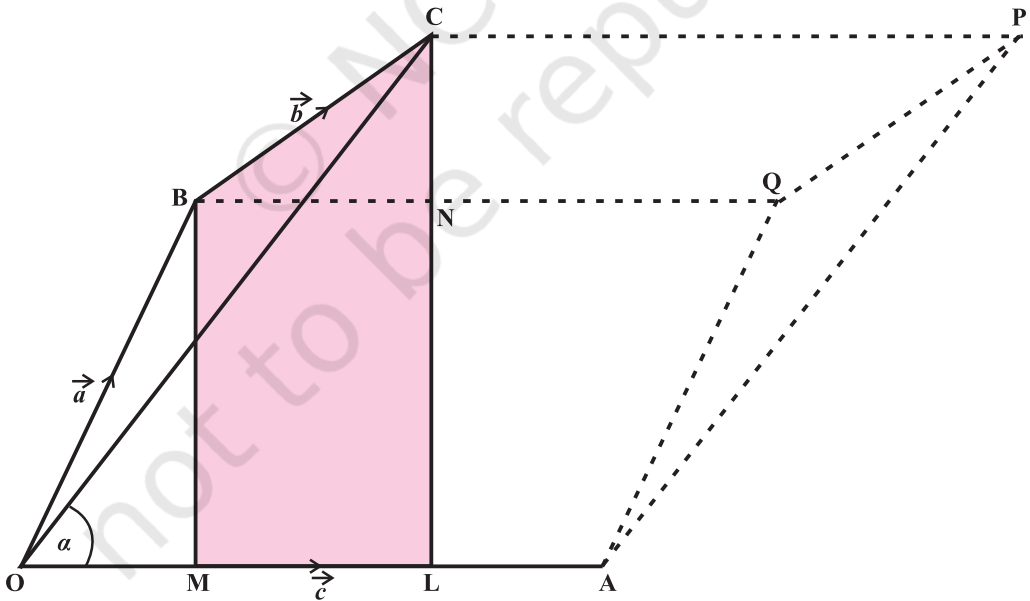

Fig. 20

4. Draw BC (= 3 cm, say) making an angle (say 30°) with $\overrightarrow{OA}$. Let $\overrightarrow{BC} = \vec{b}$

5. Draw perpendiculars BM, CL and BN.

6. Complete parallelograms OAPC, OAQB and BQPC.

DEMONSTRATION

1. $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \vec{a} + \vec{b}$, and let $\angle COA = \alpha$.

2. $\left|\vec{c} \times \left(\vec{a} + \vec{b}\right)\right| = \left|\vec{c}\right|\left|\vec{a} + \vec{b}\right| \sin \alpha$ = area of parallelogram OAPC.

3. $\left|\vec{c} \times \vec{a}\right|$ = area of parallelogram OAQB.

4. $\left|\vec{c} \times \vec{b}\right|$ = area of parallelogram BQPC.

5. Area of parallelogram OAPC = (OA) (CL)

$$= (OA) (LN + NC) = (OA) (BM + NC)$$

$$= (OA) (BM) + (OA) (NC)$$

$$= \text{Area of parallelogram OAQB} + \text{Area of parallelogram BQPC}$$

$$= \left|\vec{c} + \vec{a}\right| + \left|\vec{c} \times \vec{b}\right|$$

So, $\left|\vec{c} \times (\vec{a} + \vec{b})\right| = \left|\vec{c} \times \vec{b}\right| + \left|\vec{c} \times \vec{b}\right|$

Direction of each of these vectors $\vec{c} \times (\vec{a} + \vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ is perpendicular to the same plane.

So, $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

OBSERVATION

$\left|\vec{c}\right| = \left|\overrightarrow{OA}\right| = OA = $ _____

$\left|\vec{a}+\vec{b}\right| = \left|\overrightarrow{OC}\right| = OC = $ _____

CL = _____

$\left|\vec{c}\times(\vec{a}+\vec{b})\right| = $ Area of parallelogram OAPC

$\qquad = (OA)\ (CL) = $ _____ sq. units $\qquad$ (i)

$\left|\vec{c}\times\vec{a}\right| = $ Area of parallelogram OAQB

$\qquad = (OA)\ (BM) = $ _____ $\times$ _____ $=$ _____ $\qquad$ (ii)

$\left|\vec{c}\times\vec{b}\right| = $ Area of parallelogram BQPC

$\qquad = (OA)\ (CN) = $ _____ $\times$ _____ $=$ _____ $\qquad$ (iii)

From (i), (ii) and (iii),

Area of parallelogram OAPC = Area of parallelgram OAQB + Area of Parallelgram _____.

Thus $\quad \left|\vec{c}\times\right|(\vec{a}+\vec{b})\left|=\right|\vec{c}\times\vec{a}\left|+\right|\vec{c}\times\vec{b}\left|\right.$

$\vec{c}\times\vec{a},\ \vec{c}\times\vec{b}$ and $\vec{c}\times(\vec{a}+\vec{b})$ are all in the direction of _____ to the plane of paper.

Therefore $\quad \vec{c}\times(\vec{a}+\vec{b}) = \vec{c}\times\vec{a} + $ _____.

APPLICATION

Through the activity, distributive property of vector multiplication over addition can be explained.

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